Modifications to ACI 318 shear design method for prestressed concrete members: Detailed method

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- This study reviews and proposes improvements related to the American Concrete Institute's *Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)* shear design provisions for prestressed concrete one-way members.
- Proposed modifications for the ACI 318-19 detailed method were verified using up-to-date shear databases to evaluate analytical accuracy and safety levels.
- Three shear design examples were developed to elaborate on the proposed changes.

Through extensive research studies conducted in the past few decades, it is known that the shear strength of a concrete beam depends on various design parameters, including the following:

- compressive strength of concrete f_c'
- effective beam depth d
- shear span-to-depth ratio a/d or $M_u/V_u d$, where a is shear span, M_u is factored flexural moment, and V_u is factored shear force
- longitudinal tensile reinforcement ratio ρ_{wt}
- axial force or prestress N_u/A_g or f_{pc}, where A_g is gross area of concrete section, f_{pc} is concrete compressive stress after allowance for prestress losses, and N is factored flexural moment
- shear reinforcement ratio $\rho_v f_{yt}/\sqrt{f_c'}$, where f_{yt} is yield strength of transverse reinforcement and ρ_v is transverse reinforcement ratio

Also, the effects of size, crack surface roughness, crack width, flange, longitudinal reinforcement strain, and depth of compression zone have been under discussion. To account for intricate shear behavior, major design standards include one-way shear design provisions, especially for the shear strength provided by concrete V_c, with the differences in the standards reflecting their different theoretical backgrounds.

Eurocode 2[1] proposes integrated shear strength equations that do not distinguish between reinforced concrete and prestressed concrete. This standard suggests an empirical formula for a cracked section and a formula derived from the elastic stress distribution considering the axial force for an uncracked section.

The CSA Group's *Design of Concrete Structures* (CSA A23.3)[2] and the American Association of State Highway and Transportation Officials' *AASHTO LRFD Bridge Design Specifications*[3] also provide an integrated shear strength model for reinforced concrete and prestressed concrete one-way members, which is based on the modified compression field theory by Vecchio and Collins.[4] In this model, the concrete contribution factor β and the angle of diagonal compression field θ are the key parameters, which are determined by the strain of longitudinal tensile reinforcement ε_s under a given bending moment, shear force, axial force, and prestress. The first edition of the AASHTO LRFD specifications[5] required iterative calculations or design aid charts to obtain β and θ. Hawkins et al.[6,7] and Hawkins and Kuchma[8] have made efforts to simplify the unnecessarily complex procedures. Currently, CSA A23.3 and the AASHTO LRFD specifications provide explicit expressions for β and θ.

A similar design approach is also proposed in *fib* Model Code 2010.[9] It introduces four different levels of simplification (in other words, parameter assumptions) according to levels 1 to 4, allowing the designer to select a reasonable level of calculation convenience and prediction accuracy.

Although most design standards suggest reinforced concrete-prestressed concrete unified shear strength models, the American Concrete Institute's *Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)*[10] presents separate shear design provisions for reinforced concrete and prestressed concrete one-way members. The original ACI 318 reinforced concrete provision—which was found in editions up to ACI 318-14[11]—was complex and involved many equations (eight equations for V_c) taking the key parameters f_c', ρ_{wt}, and M_u/V_ud into account. These complexities were somewhat alleviated in ACI 318-19 through the cooperation of Joint ACI-American Society of Civil Engineers (ASCE) Committee 445 and ACI Subcommittee 318-E, as elucidated in Kuchma et al.[12] In ACI 318-19, the following changes were made:

- Eight approximated or detailed equations were reduced to three integrated equations.
- The moment and shear demand term M_u/V_ud was excluded.
- A more realistic dependency of reinforcement ratio was considered (that is, shear strength proportional to $\rho_{wt}^{1/3}$).
- The size effect coefficient λ_s was newly introduced.

While there have been minor updates in current shear design provisions for prestressed concrete one-way members since ACI 318-63,[13] as summarized chronologically in **Table 1**, the underlying design philosophy did not change until ACI 318-19. That design method also had been adopted in the *AASHTO Standard Specifications for Highway Bridges*[14] until its last edition in 2002. Whereas the shear design method for conventional reinforced concrete changed in ACI 318-19, the ACI standard still has two design methods to calculate V_c, which are appropriately called detailed and approximate methods. In the detailed method, the lesser of web shear strength V_{cw} and flexure shear strength V_{ci} is taken as V_c for prestressed concrete components. Shear strength V_{cw} was derived by using Mohr's stress circle on the web or at the junction between flange and web subjected to biaxial stress state. This approach was simplified to its current form in ACI 318-19 by Mattock.[15] Concern was raised by Kuchma et al.[16] regarding the sudden increase in strength V_{cw}, regardless of prestress level when compared with that of reinforced concrete components (from $0.17\sqrt{f_c'}$ to $0.29\sqrt{f_c'}$ MPa [$2\sqrt{f_c'}$ to $3.5\sqrt{f_c'}$ psi]). Meanwhile, the origin of V_{ci} can be traced back to Sozen and Hawkins,[17] where its semiempirical derivation process can be found. Since V_{ci} separately considers the effect of dead load V_d, including self-weight apart from externally applied loads, its computational procedures to compute all force and stress terms excluding the effect of V_d are cumbersome and difficult to code in commercial software used in practice. Kamara et al.[18] note that the exact meanings of and computational methods for each force and stress term excluding the effect of dead load V_d are still confusing when the V_{ci} equation is applied to even simple design examples.[19,20]

To simplify the V_{ci} equation, ACI 318 has included since the 1971 edition an alternative method (the approximate method specified in Table 22.5.6.2 of ACI 318-19), based on a 1970 proposal by MacGregor and Hanson.[21] However, application of the approximate method is strictly limited to situations when the effective prestressing force f_{se} times the area of prestressed reinforcement A_{ps} is greater than 40% of the tensile capacity provided by all the longitudinal reinforcements.

Professionals working with prestressed concrete industry, in design and construction practices and academia, have pointed out that the current shear provisions require considerable computational efforts due to interrelated parameters such as design forces and section properties, particularly for V_{ci}.[18,19] The criticism demonstrates why modification of the prestressed concrete shear provision is desirable.

This paper describes a study, which was briefly reported in ACI's *Concrete International*,[22] that sought to improve upon the ACI 318 shear design provisions for prestressed concrete components. The aim was to retain the ACI 318 design philosophy as well as the safety levels related to analytical accuracy inherent in ACI 318 while expanding the applicability of the design provisions. The paper begins with a review of the detailed method of current ACI shear design approach, followed by discussion of technical issues raised by professionals during the derivation process. Proposed modifications for the ACI 318 detailed method were

Table 1. History of shear design expressions for prestressed concrete members in American Concrete Institute's *Building Code Requirements for Structural Concrete (ACI 318)*

Edition	Shear design expressions	Edition	Shear design expressions
ACI 318-63	$V_{ci} = 0.05\sqrt{f'_c}b'd + \frac{M_{cr}}{\frac{M}{V} - \frac{d}{2}} + V_d$ $V_{cw} = \left(0.29\sqrt{f'_c} + 0.3f_{pc}\right)b'd + V_p$	ACI 318-71*	$V_c = \left(0.05\sqrt{f'_c} + 4.8\frac{V_u d}{M_u}\right)b_w d$ $V_{ci} = 0.05\sqrt{f'_c}b_w d + V_d + \frac{V_i M_{cr}}{M_{max}}$ $V_{cw} = \left(0.29\sqrt{f'_c} + 0.3f_{pc}\right)b_w d + V_p$
ACI 318-02	$V_c = \left(0.05\sqrt{f'_c} + 4.8\frac{V_u d}{M_u}\right)b_w d$ $V_{ci} = 0.05\sqrt{f'_c}b_w d + V_d + \frac{V_i M_{cr}}{M_{max}}$ $V_{cw} = \left(0.29\sqrt{f'_c} + 0.3f_{pc}\right)b_w d + V_p$	ACI 318-05†	$V_c = \left(0.05\sqrt{f'_c} + 4.8\frac{V_u d_p}{M_u}\right)b_w d$ $V_{ci} = 0.05\sqrt{f'_c}b_w d_p + V_d + \frac{V_i M_{cre}}{M_{max}}$ $V_{cw} = \left(0.29\sqrt{f'_c} + 0.3f_{pc}\right)b_w d_p + V_p$
ACI 318-08, 318-11, and 318-14	$V_c = \left(0.05\sqrt{f'_c} + 4.8\frac{V_u d_p}{M_u}\right)b_w d$ $V_{ci} = 0.05\sqrt{f'_c}b_w d_p + V_d + \frac{V_i M_{cre}}{M_{max}}$ $V_{cw} = \left(0.29\sqrt{f'_c} + 0.3f_{pc}\right)b_w d_p + V_p$	ACI 318-19‡	$V_c = \left(0.05\lambda\sqrt{f'_c} + 4.8\frac{V_u d_p}{M_u}\right)b_w d \le \left(0.05\lambda\sqrt{f'_c} + 4.8\right)b_w d$ $0.17\lambda\sqrt{f'_c}b_w d \le V_c \le 0.42\lambda\sqrt{f'_c}b_w d$ $V_{ci} = 0.05\lambda\sqrt{f'_c}b_w d_p + V_d + \frac{V_i M_{cre}}{M_{max}}$ For components with $A_{ps}f_{se} < 0.4(A_{ps}f_{pu} + A_s f_y)$, $V_{ci} \ge 0.14\lambda\sqrt{f'_c}\, b_w d_p$ For components with $A_{ps}f_{se} \ge 0.4(A_{ps}f_{pu} + A_s f_y)$, $V_{ci} \ge 0.17\lambda\sqrt{f'_c}\, b_w d_p$ $V_{cw} = (0.29\lambda\sqrt{f'_c} + 0.3f_{pc})b_w d_p + V_p$

Note: Equations are for use with International System units. For f'_c in MPa, use $\sqrt{f'_c}$. For f'_c in psi, use $12\sqrt{f'_c}$. A_{ps} = area of prestressed longitudinal tension reinforcement; A_s = area of nonprestressed longitudinal tension reinforcement; b'= minimum width of web of a flanged component according to ACI 318-63; b_w = web width of component; d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement; d_p = distance from extreme compression fiber to prestressed longitudinal reinforcement; f'_c = compressive strength of concrete; f_{pc} = compressive stress in concrete after allowance for all prestress losses; f_{pu} = tensile strength of prestressed reinforcement; f_{se} = effective prestress in prestressed reinforcement; f_y = yield strength of nonprestressed longitudinal reinforcement; M = bending moment due to externally applied load, according to ACI 318-63; M_{cr} = cracking moment; M_{cre} = moment causing flexural cracking due to externally applied loads; M_{max} = maximum factored moment at section due to externally applied loads; M_u = factored flexural moment; V = shear force due to externally applied load, according to ACI 318-63; V_c = nominal shear strength provided by concrete; V_{ci} = flexure shear strength; V_{cw} = web shear strength; V_d = shear force at section due to unfactored dead load; V_i = factored shear force; V_p = vertical component of effective prestress force; V_u = factored shear force at section; λ = lightweight concrete factor. 1 MPa = 0.145 ksi.

*MacGregor introduced approximate method.

†d_p first appeared in the code, and M_{cr} was modified to M_{cre}.

‡The minimum limit of V_{ci} has been updated depending on prestress level; the ACI 318 detailed method was adopted in strength estimations of prestressed components, and $\sqrt{f'_c}$ is limited to 8.4 MPa, if the minimum shear reinforcement is not provided. For pretensioned component, transfer length is considered as 50 times the diameter of prestressing strand in the end region.

verified by using up-to-date shear databases to evaluate analytical accuracy and safety levels. Finally, three shear design examples were developed to elaborate on the proposed changes.

Review of the ACI 318-19 detailed shear design method

General requirements

According to ACI 318-19,[10] the nominal shear strength of prestressed concrete one-way members V_n can be estimated as the sum of contributions of concrete V_c and stirrup V_s (Eq. [1] and [2]). The designer can calculate V_c using either the detailed or approximate method.

For f'_c in MPa,

$$V_n = V_c + V_s \le V_c + 0.66\sqrt{f'_c}\, b_w d \tag{1}$$

where

b_w = web width of component

For f_c' in psi,

$$V_n = V_c + V_s \le V_c + 8\sqrt{f_c'}\,b_w d \tag{2}$$

As suggested by Kuchma et al.,[16] it may be reasonable to use Eq. (3) to define d, based on the centroid of resultant tension force, which needs not be taken less than $0.8h$, where h is member height or thickness, in accordance with ACI 318-19.

$$d = \frac{A_s f_s d_s + A_p f_{py} d_p}{A_s f_s + A_p f_{py}} \tag{3}$$

where

A_s = area of nonprestressed longitudinal tension reinforcement

f_y = yield strength of nonprestressed longitudinal reinforcement

d_p = distance from extreme compression fiber to prestressed longitudinal reinforcement

d_s = distance from extreme compression fiber to centroid of nonprestressed longitudinal reinforcement

A_{ps} = area of prestressed longitudinal tension reinforcement

f_{py} = yield strength of prestressed reinforcement

Detailed method

The detailed method specified in ACI 318-19 defines shear contribution provided by concrete V_c in prestressed concrete components as the lesser of V_{ci} and V_{cw}. The former failure mechanism develops when flexural cracking propagates and merges into inclined shear cracking in the shear span. No major changes to the flexure shear strength equation V_{ci} have been introduced since it was first introduced into ACI 318 in 1963 (Table 1).

For f_c' in MPa,

$$V_{ci} = 0.05\lambda\sqrt{f_c'}b_w d_p + V_d + \frac{V_i M_{cre}}{M_{\max}} \tag{4}$$

where

λ = lightweight concrete factor

V_i = factored shear force at section due to externally applied loads occurring simultaneously with M_{max}

M_{max} = maximum factored moment at section due to externally applied loads

M_{cre} = moment causing flexural cracking due to externally applied loads

For f_c' in psi,

$$V_{ci} = 0.6\lambda\sqrt{f_c'}b_w d_p + V_d + \frac{V_i M_{cre}}{M_{\max}} \tag{5}$$

For f_c' in MPa, when $A_{ps} f_{se} \ge 0.4(A_{ps} f_{pu} + A_s f_y)$,

$V_{ci} \ge 0.17\lambda\sqrt{f_c'}\,b_w d$

where

f_{pu} = tensile strength of prestressed reinforcement

For f_c' in psi,

$V_{ci} \ge 2\lambda\sqrt{f_c'}\,b_w d$

For f_c' in MPa, when $A_{ps} f_{se} < 0.4(A_{ps} f_{pu} + A_s f_y)$,

$V_{ci} \ge 0.14\lambda\sqrt{f_c'}\,b_w d$

For f_c' in psi,

$V_{ci} \ge 1.7\lambda\sqrt{f_c'}\,b_w d$

For f_c' in MPa,

$$M_{cre} = \frac{I}{y_t}\left(0.5\lambda\sqrt{f_c'} + f_{pe} - f_d\right) \tag{6}$$

where

f_d = compressive stress due to unfactored dead load at extreme fiber of section where tensile stress is caused by externally applied loads

f_{pe} = compressive stress in concrete due to effective prestress forces only

y_t = distance from centroidal axis to top surface of gross (composite) section

For f_c' in psi,

$$M_{cre} = \frac{I}{y_t}\left(6\lambda\sqrt{f_c'} + f_{pe} - f_d\right) \tag{7}$$

The first term in Eq. (4) and (5)—that is, $0.05\lambda\sqrt{f_c'}\,b_w d_p$ for f_c' in MPa and $0.6\lambda\sqrt{f_c'}\,b_w d_p$ for f_c' in psi—represents the additional shear force required to transform flexural cracks into a critical shear crack. It was empirically obtained based on past test observations, taking into account the tendency proportional to the square root of concrete compressive strength $\sqrt{f_c'}$.[17] The last two terms in Eq. (4) and (5) indicate the shear force existing at the time and location of flexural cracking: V_d is the shear force due to dead load and $V_i M_{cre}/M_{max}$ is the shear force due to externally added loads, which cause the tensile stress in the extreme tensile fiber to reach $0.5\lambda\sqrt{f_c'}$ for f_c' in MPa and $6\lambda\sqrt{f_c'}$ for f_c' in psi.

Recent investigations identified other critical influential factors that affect the shear capacity of prestressed concrete beams.[23–26] These factors include the longitudinal reinforcement ratio ρ_{wt}. The method in ACI 318-19 does not consider ρ_{wt}, but this study considers it in the modification. Also, interdependent variables related to design forces and section properties make design cumbersome for the calculation of V_{ci}, which provides a motivation to improve the current design equation.

Web shear strength V_{cw} is estimated as the point where the principal tensile stress in the centroid of the section reaches the tensile strength of concrete. The equation for V_{cw} is quite straightforward and its basis relies on the theory of elasticity.[27–29] In ACI 318-19, the web shear strength of prestressed concrete beams is calculated as

For f'_c in MPa,

$$V_{cw} = (0.29\lambda\sqrt{f'_c} + 0.3f_{pc})b_w d_p + V_p \tag{8}$$

where

V_p = vertical component of effective prestress force

For f'_c in psi,

$$V_{cw} = (3.5\lambda\sqrt{f'_c} + 0.3f_{pc})b_w d_p + V_p \tag{9}$$

Note that ACI 318-19 section 22.5.6.3.3 allows taking V_{cw} as the shear force resulting in a principal tensile stress of $0.33\lambda\sqrt{f'_c}$ in MPa and $4\lambda\sqrt{f'_c}$ in psi). The tensile strength of concrete can vary between $0.17\sqrt{f'_c}$ and $0.29\sqrt{f'_c}$ in MPa ($2\sqrt{f'_c}$ to $3.5\sqrt{f'_c}$ in psi), depending on the level of prestress and axial compression, if any. The web shear strength presented in Eq. (8) and (9) implies an abrupt increase in concrete shear strength even at low levels of prestress and a variable angle of shear cracking (that is, not 45 degrees). This inconsistency between the reinforced concrete and prestressed concrete design equations needs to be reconsidered in subsequent ACI modifications and is the study subject in this research.

Shear database

This study used observed data from prestressed concrete beam shear tests to verify the ACI 318-19 shear design methods and the proposed modifications. Prestressed concrete beam failure in shear has drawn the research community's attention since the beginning of the 21st century.[30] Collected test results are often used for the evaluation of various design code models.[20] Results cover a full range of practical experimental data, including various geometries, material properties, and loading and boundary conditions. This study primarily used the ACI-DAfStb database, which has been officially established by joint ACI-ASCE Committee 445 and German Committee of Reinforced Concrete[31] for reinforced concrete and prestressed concrete test specimens. Two major groups in the database were used in this study: prestressed concrete beams and extruded hollow-core slabs, with the prestressed concrete beams further divided into those with and without shear reinforcement. All of the hollow-core slab specimens collected for evaluation were not reinforced in shear because of their unique production process (the extrusion method). The study excluded prestressed concrete specimens with fiber-reinforced polymer shear reinforcement,[32,33] external prestressing,[34] or self-consolidating concrete.[35] Also excluded were one-way components with steel-concrete composite,[36] steel fibers,[37] recycled concrete aggregates,[38] or impact load,[39] as well as two-way components.[40] For the evaluation database, the filtration criteria included the following:

- compressive strength of concrete f'_c equal to or greater than 12 MPa (1740 psi)
- shear span-to-depth ratio a/d equal to or greater than 2.4
- member height h equal to or greater than 70 mm (2.76 in.)
- web width b_w equal to or greater than 50 mm (1.97 in.)
- ratio between shear strength and shear force estimated at the flexural strength β_{flex} less than or equal to 1.1

The final version of the ACI-DAfStb database used in this study contained 332 shear test results for prestressed concrete beams with a straight tendon profile subjected to point load(s), with the specimens divided into 118 with shear reinforcement and 214 with no shear reinforcement (**Fig. 1**). The specimens in the data set can also be classified as 163 pretensioned beams and 169 post-tensioned beams. The database also included unbonded post-tensioned beams and variations in section shapes, such as rectangular, bulb tee, and tee.

The compressive strength of collected test specimens ranged from 12.9 to 102.9 MPa (1870 to 14,924 psi). The compressive strengths for most of the prestressed concrete beam specimens were between 30 and 50 MPa (4350 and 7250 psi) (Fig. 1). The effective depth d of the specimens, which was calculated by using Eq. (3), ranged for the most part from 150 to 350 mm (6 to 14 in.). In a few cases, effective depth d up to roughly 1363 mm (54 in.) was also included. Of the 332 prestressed concrete beams included in the database, 184 fully prestressed members with no bonded nonprestressed reinforcement were included.

To address impacts of modifications on the hollow-core slab industry, pretensioned hollow-core slab specimens (**Fig. 2**) produced by dry-casting (extrusion method) from Park et al.[27] and Lee et al.[28] were compiled with the ACI-DAfStb shear database. A total of 145 prestressed hollow-core slab specimens were added. None of these specimens included a cast-in-place concrete topping. The collected hollow-core slab specimens were also not reinforced in shear. Their concrete compressive strengths ranged from 40 to 114 MPa (5802 to 16,534 psi), and the vast majority had concrete compressive strengths between 50 and 70 MPa (7250 and 10,150 psi). All the specimens reported in hollow-core slab test database[27,28] failed due to web shear cracking.

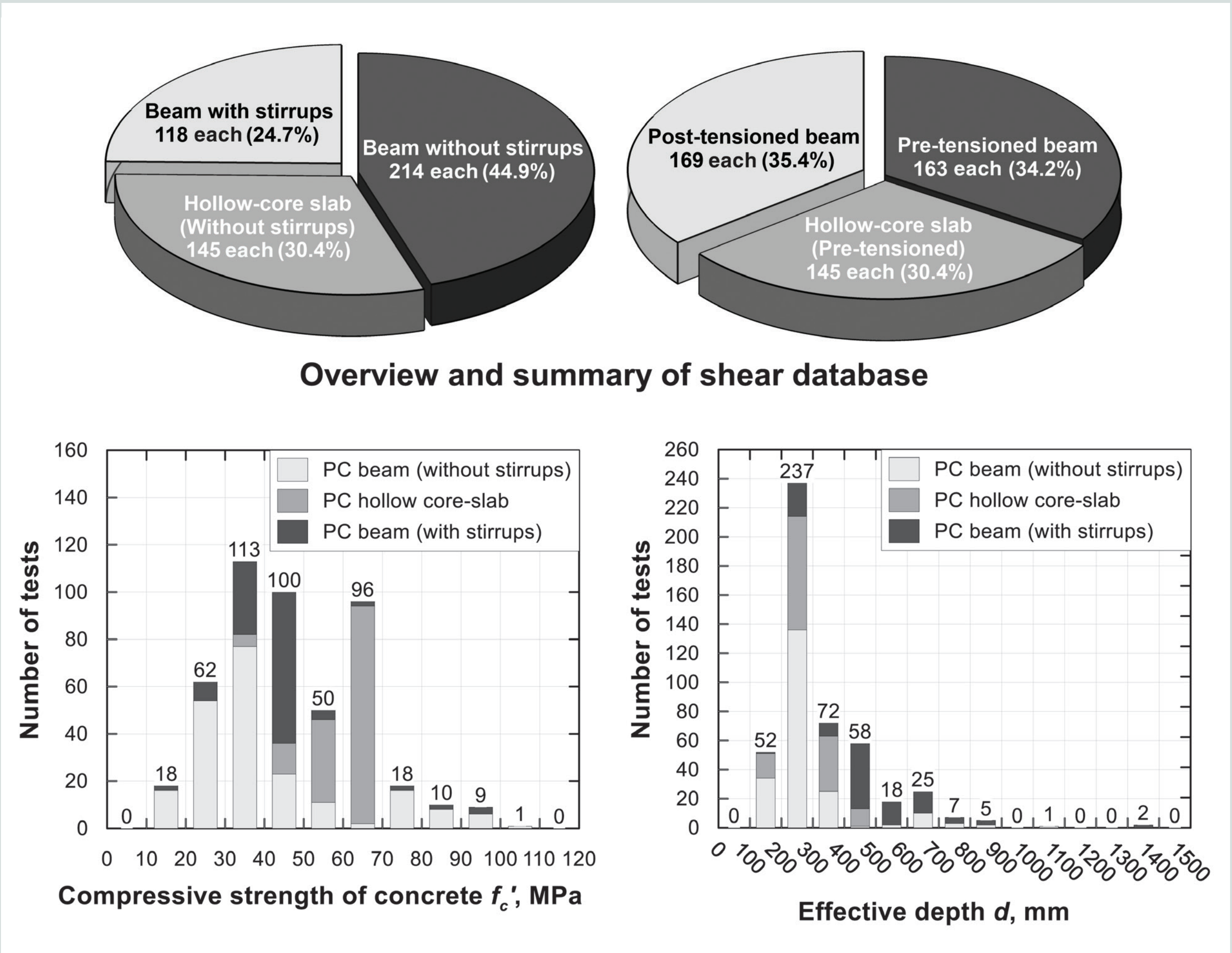

Figure 1. Distribution of key influential factors in shear database. Note: PC = prestressed concrete. 1 mm = 0.0394 in.; 1 MPa = 0.145 ksi.

Figure 2. Hollow-core slabs produced by extrusion method.

Per ACI 318-19 section 7.6.3.1, minimum shear reinforcement should be provided if the factored shear force V_u exceeds half of the design web shear strength $0.5\phi V_{cw}$, where ϕ is the strength reduction factor, for prestressed hollow-core slab components with an untopped depth exceeding 315 mm (12.5 in.). In other words, the minimum shear reinforcement provision can be directly interpreted for web shear capacity of hollow-core slab components and V_{cw} of hollow-core slab members is taken as half of the strength calculated by Eq. (8) and (9) in this study. Of the collected hollow-core slab specimens, 41% (60 specimens) should have been subjected to this rule because those specimens had a component height greater than 315 mm (12.5 in.). However, it should be noted that ACI 318-19 section 7.6.3.2 allows shear strength evaluation by testing, and that method is preferred in practice.

The prestressed concrete beam and hollow-core slab database for verification consisted of a total of 477 specimens. Only 9 specimens (2%) were over-reinforced in flexure (that is, they had a compression-controlled section per ACI 318-19 section 21.2.2), whereas 435 specimens (91%) had a tension-controlled section. As most specimens were under-reinforced in flexure, the database specimens are considered to reasonably represent design components in practice.

Modification of the shear design method

Flexural shear strength V_{ci}

The first modification of V_{ci} is linked to the effect of dead load V_d. MacGregor and Hanson[21] proposed the removal of V_d to simplify the calculation process and this change was made to the ACI 318 method in ACI 318-71 (Table 1). For the purpose of simplicity, the influence of V_d in the equation for flexure shear strength described in Eq. (4) and (5) may likewise be eliminated. The contribution of V_d to V_{ci} of all prestressed concrete beams and hollow-core slabs (total 477 specimens) is shown in **Fig. 3**. The contribution of V_d is up to 7.3% of the shear capacity provided by the shear-flexure equation V_{ci} specified in ACI 318-19. For most of the test specimens, the V_d contribution ranged from 0.5% to 3.5%, with a mean value of 1.58%. By observation, the influence of dead load effect V_d is marginal and can be disregarded for simplicity with little impact in terms of accuracy. A similar expression is presented in Eq. (R22.5.6.3.1.d) of ACI 318-19 section R22.5.6.3.1 for noncomposite prestressed concrete beams subjected to uniformly distributed loads.

By removing V_d from Eq. (4) and (5), M_{cre} in Eq. (6) and (7) is no longer affected by the dead load and, likewise, stress due to unfactored dead load at the extreme fiber of section f_d can also be removed. Subsequently, M_{cre} may be replaced with the following equation for cracking moment M_{cr}, which is consistent with the flexural cracking moment typically used in serviceability design.

For f_c' in MPa,

$$M_{cr} = \left(0.63\lambda\sqrt{f_c'} + f_{pe}\right)\frac{I}{y_t}$$

where

I = moment of inertia of section about centroidal axis

For f_c' in psi,

$$M_{cr} = \left(7.5\lambda\sqrt{f_c'} + f_{pe}\right)\frac{I}{y_t}$$

Figure 3 shows the ratios between V_iM_{cre}/M_{max} and its simplified form V_uM_{cr}/M_u for the test specimens in the shear database against the normalized magnitude of beneficial compressive stress induced in extreme concrete fiber due to prestress f_{pe}/f_c'. As f_{pe} increases, V_uM_{cr}/M_u approaches V_iM_{cre}/M_{max}. Even for very small magnitude cases of f_{pe} (such as partial prestressing), removal of the dead load effect causes a 10% difference on the unconservative side when compared to the original. However, this effect is counteracted by excluding V_d from the shear strength. By replacing M_{max} and V_i with factored design forces M_u and V_u, computation of V_{ci} becomes more intuitive and simpler.

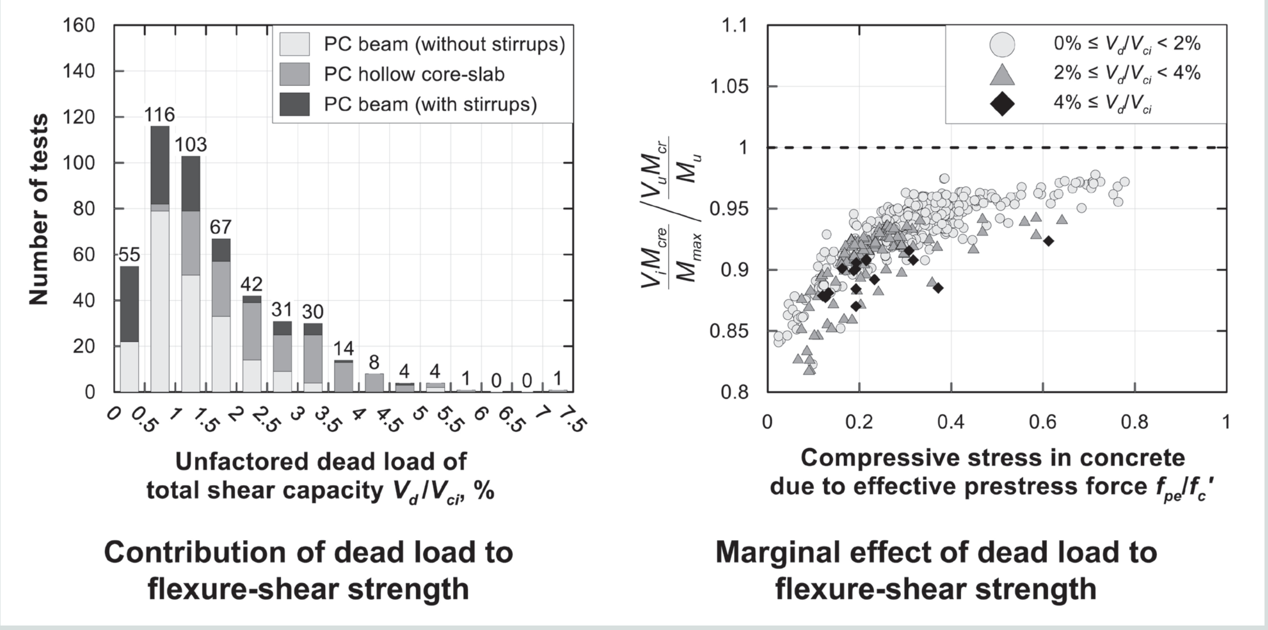

Figure 3. Effect of modification on flexure-shear capacity. Note: f_{ci}' = compressive strength of concrete; f_{pe} = compressive stress in concrete due to effective prestress forces only; M_{cr} = cracking moment; M_{cre} = moment causing flexural cracking due to externally applied loads; M_{max} = maximum factored moment at section due to externally applied loads; M_u = factored flexural moment; PC = prestressed concrete; V_{ci} = flexure-shear strength; V_d = shear force at section due to unfactored dead load; V_i = factored shear force at section due to externally applied loads occurring simultaneously with M_{max}; V_u = factored shear force at section.

The first term of Eq. (4) and (5) is empirically derived as a function of compressive strength of concrete f'_c based on test observations until 1962.[17] Though this formula predicts flexural-shear strength accurately, many recent investigations[24,25,41–46] reveal that the shear capacity of prestressed concrete members is also significantly affected by the longitudinal reinforcement ratio ρ_{wt}, which is defined as:

$$\rho_{wt} = \rho_w + \rho_{pw} = (A_s + A_{ps})/(b_w d)$$

where

ρ_{pw} = prestressed longitudinal reinforcement ratio

ρ_w = nonprestressed longitudinal reinforcement ratio

To evaluate this effect, 111 samples were selected from the shear database where the mode of shear failure was clearly reported as flexure shear.[47–53] To evaluate the flexure shear strength for the selected test data, a critical section was assumed to be $h/2$ apart from the loading point (that is, a-$h/2$ from the support) for all flexure-shear failed specimens. **Figure 4** shows the influence of ρ_{wt} on shear capacity in line with a normalized shear contribution of the simplified term (that is, $V_u M_{cr}/M_u b_w d_p \sqrt{f'_c}$).[17] As ρ_{wt} increases, the conservatism of Eq. (4) and (5)—where V_d is eliminated, M_{cre} is replaced with M_{cr}, and M_{max} and V_i are replaced with factored design forces M_u and V_u—becomes more obvious.

To reflect the effect of ρ_{wt} properly, a longitudinal reinforcement coefficient K (where K is $4(\rho_{wt})^{1/3}$) is introduced. According to ACI 318-19 section 22.5.5.1, for nonprestressed reinforced concrete members with minimum shear reinforcement ($A_v \geq A_{v,min}$ where A_v is area of shear reinforcement and $A_{v,min}$ is minimum area of shear reinforcement), V_c is defined by either an approximate formula (Eq. [10]) or a detailed formula (Eq. [11]). The only difference between the formulas is the coefficient (either 0.17 or $0.66[\rho_{wt}]^{1/3}$ with f'_c in MPa). Because Eq. (10) is a lower bound of Eq. (11), $4(\rho_{wt})^{1/3}$ does not need to be less than 1.0 (that is, $K \geq 1.0$). The same rationale behind the reinforced concrete shear provision in ACI 318-19 is applied to prestressed concrete equations. Also, the effect of longitudinal reinforcement ratio can be evaluated more intuitively by introducing the coefficient K.

$$V_c = \left[0.17\lambda\sqrt{f'_c} + \frac{N_u}{6A_g}\right] b_w d \quad (10)$$

$$V_c = \left[0.66\lambda\rho_{wt}^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right] b_w d \quad (11)$$

For f'_c in MPa,

$$V_{ci} = 0.05\lambda K\sqrt{f'_c} b_w d_p + \frac{V_u M_{cr}}{M_u} \quad (12)$$

For f'_c in psi,

$$V_{ci} = 0.6\lambda K\sqrt{f'_c} b_w d_p + \frac{V_u M_{cr}}{M_u} \quad (13)$$

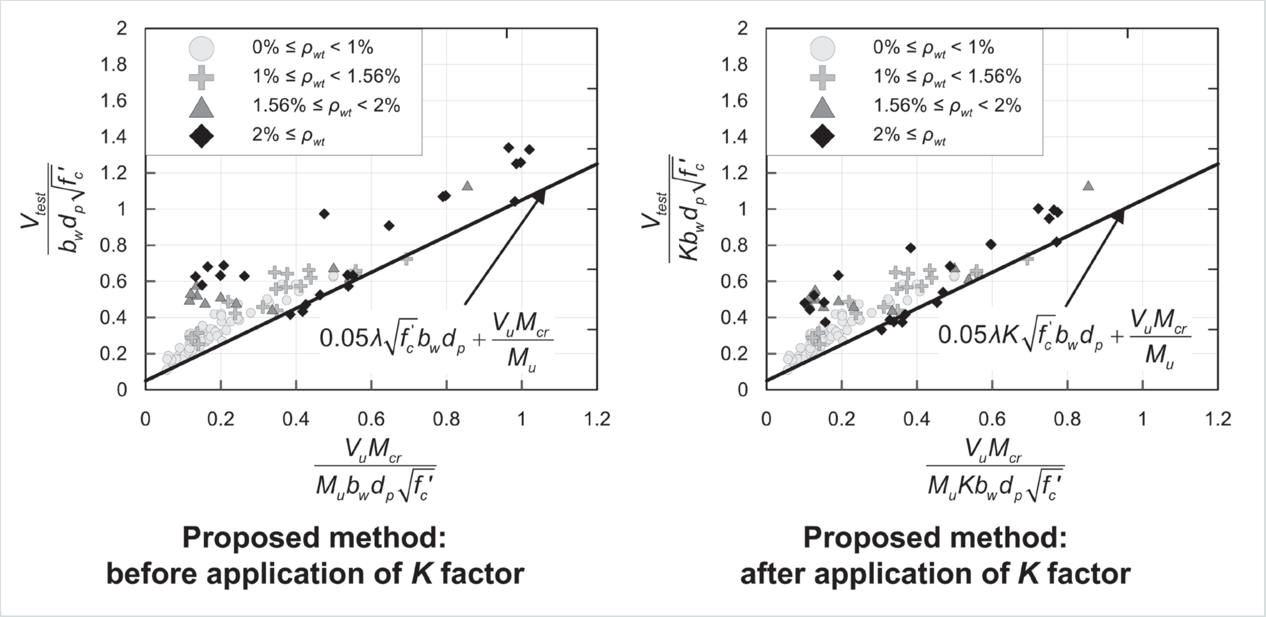

Figure 4. Effect of longitudinal reinforcement ratio on flexure-shear strength. Note: b_w = web width of component; d_p = distance from extreme compression fiber to prestressed longitudinal reinforcement; f'_{ci} = compressive strength of concrete; K = longitudinal reinforcement coefficient; M_{cr} = cracking moment; M_u = factored flexural moment; V_{test} = measured shear strength in shear database; V_u = factored shear force at section; λ = lightweight concrete factor; ρ_{wt} = longitudinal reinforcement ratio.

A comparison of the graphs in Fig. 4 shows that the coefficient K has no significant impact on the overall analytical accuracy. The underestimation of shear strength for beam specimens with a high longitudinal reinforcement ratio (ρ_{wt} greater than 2.0%) is somewhat alleviated, whereas lightly reinforced beams are not affected by this modification if K equals 1.0 when ρ_{wt} is less than 1.56%.

Use of the modified V_{ci} presented in Eq. (12) and (13) was verified by comparing results with the shear database as well as the ACI 318 method. Since hollow-core slabs failed in web shear, as presented by Park et al.[27] and Lee et al.,[28] those specimens were excluded in the verification of V_{ci}.

Tables 2 and **3** present statistical values of the strength ratios between estimated and observed shear strengths (V_{test}/V_{cal} where V_{cal} is calculated shear strength and V_{test} is measured shear strength from shear database). The method expressed in Eq. (12) and (13) conservatively estimates the shear strengths of specimens regardless of prestressing method or section shape. The modified model also showed comparable results with those in ACI 318-19.

Web shear strength V_{cw}

For the test specimens with failure mode reported as web shear in the database (41 prestressed concrete beams and

Table 2. Verification of proposed modifications for specimens with no shear reinforcement

$\frac{V_{test}}{V_{cal}}$	Flexure shear		Flexure shear (hollow-core slab excluded)		Web shear (hollow-core slab excluded)		Web shear* (hollow-core slab excluded)		Detailed method	
	ACI 318	Modified model	ACI 318	Modified model	ACI 318	Modified model	ACI 318	Modified model	ACI 318	Modified model
Average	1.20	1.12	1.52	1.40	1.03	1.03	1.60	1.67	1.59	1.57
Standard deviation	0.61	0.53	0.58	0.49	0.35	0.36	0.56	0.60	0.54	0.52
Coefficient of variation	0.51	0.48	0.38	0.35	0.34	0.35	0.35	0.36	0.34	0.33
Predicted results falling below experimental data of analysis results, %	56.5	50.7	88.3	79.9	57.0	52.3	89.0	89.6	94.2	92.5
Number of test samples	359		214		214		145		359	

Note: ACI 318 = *American Concrete Institute's Building Code Requirements for Structural Concrete*; V_{cal} = calculated shear strength per ACI 318-19 or proposed detailed method; V_{cw} = web shear strength; V_{test} = measured shear strength in shear database.

* $V_{cw}/2$ for hollow-core slab components with untopped depth greater than 12.5 in. (315 mm).

Table 3. Verification of proposed modifications for specimens with shear reinforcement

$\frac{V_{test}}{V_{cal}}$	Flexure shear strength		Web shear strength		Detailed method	
	ACI 318	Modified model	ACI 318	Modified model	ACI 318	Modified model
Average	1.15	1.09	1.22	1.19	1.35	1.30
Standard deviation	0.35	0.29	0.29	0.26	0.30	0.25
Coefficient of variation	0.31	0.26	0.24	0.22	0.22	0.20
Predicted results falling below experimental data of analysis results, %	65.3	59.3	76.1	75.2	92.4	89.8
Number of test samples	118					

Note: ACI 318 = *American Concrete Institute's Building Code Requirements for Structural Concrete*; V_{cal} = calculated shear strength per ACI 318-19 or proposed detailed method; V_{test} = measured shear strength in shear database.

145 hollow-core slabs), the normalized shear strength with respect to f_{pc}/f_c' is shown in **Fig. 5**. The web shear strength is evaluated at a critical section assumed to be $h/2$ apart from the support. The black dashed line and the solid gray line indicate $0.29\sqrt{f_c'} + 0.3f_{pc}$ and $0.17\sqrt{f_c'} + 0.3f_{pc}$ in MPa ($3.5\sqrt{f_c'} + 0.3f_{pc}$ and $2\sqrt{f_c'} + 0.3f_{pc}$ in psi), respectively. Per ACI 318-19, the tensile strengths of concrete associated with prestressed concrete and reinforced concrete members are $0.29\sqrt{f_c'}$ and $0.17\sqrt{f_c'}$ in MPa ($3.5\sqrt{f_c'}$ and $2\sqrt{f_c'}$ in psi), respectively. As noted by Kuchma et al.,[16] said conjecture leads to a sudden increase in web shear capacity even for low levels of prestress—that is, even $f_{pc} \approx 0$, $V_{cw} = 0.29\sqrt{f_c'}b_w d_p$ with f_c' in MPa or $3.5\sqrt{f_c'}b_w d_p$ with f_c' in psi. The increase in shear capacity for prestressed concrete members compared with that of reinforced concrete members is as much as 70%, even when the prestressing effect $0.3f_{pc}$ is not taken into account (that is, $0.29\sqrt{f_c'}/0.17\sqrt{f_c'} \approx 1.7$ or $3.5\sqrt{f_c'}/2\sqrt{f_c'} \approx 1.7$). Even while the overall trend of web shear capacities is appropriately captured in $0.29\sqrt{f_c'} + 0.3f_{pc}$ in MPa or $3.5\sqrt{f_c'} + 0.3f_{pc}$ in psi (the ACI 318-19 expression of V_{cw}), the current formula overestimates the shear strength of prestressed concrete members with a compressive strength greater than 50 MPa (7.25 ksi). Thus, such inconsistency between the reinforced concrete and prestressed concrete shear strength equations can lead to the overestimation of strength for low prestress (Fig. 5), leading to an unsafe component design.

Within ACI 318-19, the effect of the longitudinal reinforcement ratio ρ_{wt} for reinforced concrete members is also reflected in web shear strength modification. Its non-negligible effect was also confirmed by Saqan and Frosch.[54] Figure 6 shows an increasing trend on web shear capacity depending on ρ_{wt}. To capture longitudinal reinforcement ratio ρ_{wt} effects on prestressed concrete members, the coefficient K is applied in modification of the web shear strength model in line with flexural shear strength.

Decreasing the coefficient from 0.29 to 0.17 MPa (4.2 to 2.5 psi) can lead to an unnecessarily conservative estimate (solid gray line and black dashed line in **Fig. 6**) compared with the ACI 318-19 method. Also, the web shear strength of prestressed concrete components is a function of f_{pc} in ACI 318-19. However, as shown in Fig. 6, the slope when expressed on a logarithmic scale reveals $\sqrt{f_{pc}}$ is much closer with the data trend than f_{pc}. Considering the proper level of conservatism and dependency on $\sqrt{f_{pc}}$, the web shear strength can be modified as follows:

For f_c' in MPa,

$$V_{cw} = \left(0.17\lambda K\sqrt{f_c'} + \sqrt{f_{pc}}\right)b_w d_p + V_p \quad (14)$$

For f_c' in psi,

$$V_{cw} = \left(2\lambda K\sqrt{f_c'} + 12\sqrt{f_{pc}}\right)b_w d_p + V_p \quad (15)$$

It should be noted that because the proposed Eq. (14) and (15) have a reduced portion of concrete tensile strength term, they generally tend to give a more conservative V_{cw} estimation for high-strength concrete components compared

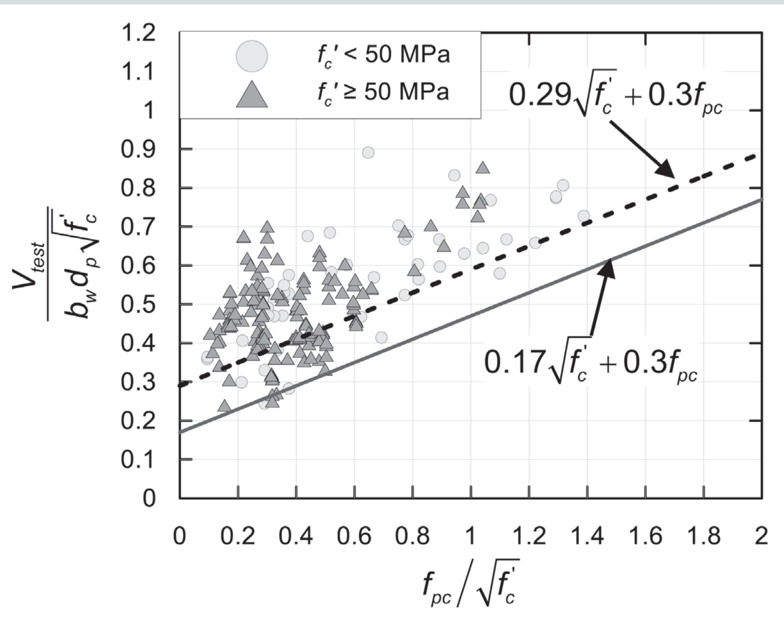

Figure 5. Influence of effective prestress on web-shear strength. Note: For f_{ci}' in MPa, use $\sqrt{f_c'}$, and for f_{ci}' in psi, use $12\sqrt{f_c'}$. b_w = web width of component; d_p = distance from extreme compression fiber to prestressed longitudinal reinforcement; f_{ci}' = compressive strength of concrete; f_{pc} = compressive stress in concrete after allowance for all prestress losses at centroid of cross section; V_{test} = measured shear strength in shear database. 1 MPa = 0.145 ksi.

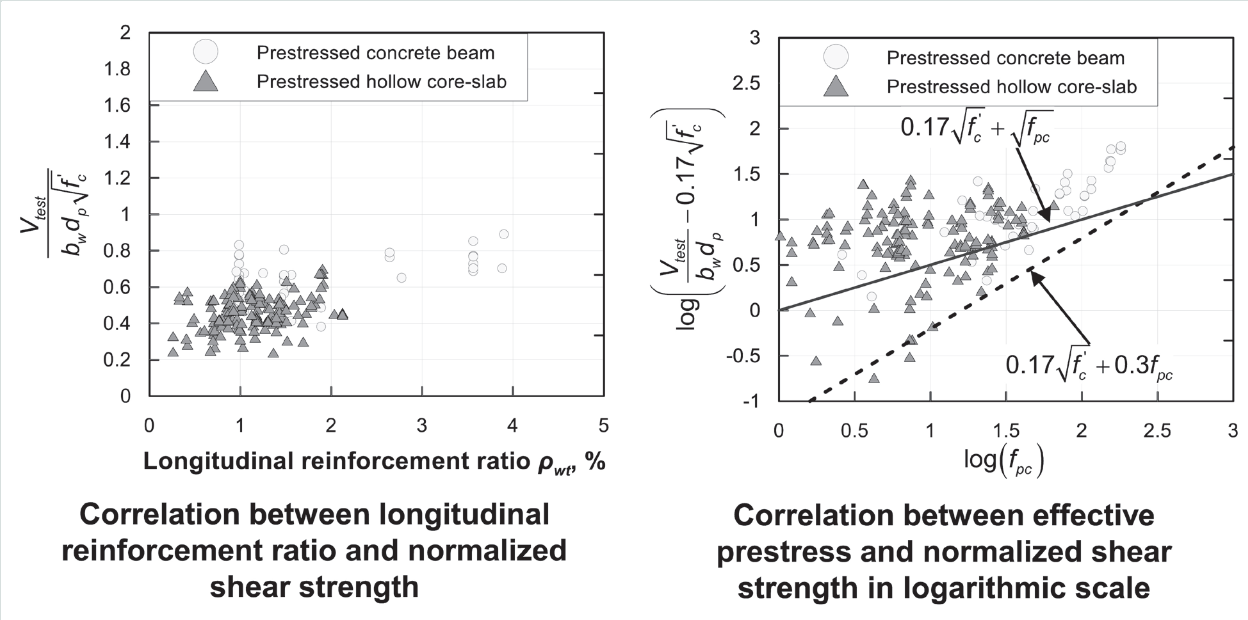

Figure 6. Key influential parameters on web-shear strength: prestress and longitudinal reinforcement ratio. Note: For f_{ci}' in MPa, use $\sqrt{f_c'}$. Note: For f_{ci}' in psi, use $12\sqrt{f_c'}$. b_w = web width of component; d_p = distance from extreme compression fiber to prestressed longitudinal reinforcement; f_{ci}' = compressive strength of concrete; f_{pc} = compressive stress in concrete after allowance for all prestress losses at centroid of cross section; V_{test} = measured shear strength in shear database; ρ_{wt} = longitudinal reinforcement ratio. 1 MPa = 0.145 ksi.

with the current formula (Eq. [4]). **Figure 7** shows if a component with zero vertical prestress is assumed (V_n is 0), the web shear strength predicted by the proposed method $V_{cw,proposed}$ tends to be more conservative than that of the current method $V_{cw,current}$ with increasing concrete compressive strength f_c'. The proposed method is more conservative regardless of the level of prestress f_{pc} for higher concrete strength ($f_c' \geq 50$ MPa [7250 psi]). However, those changes can be justified by the current method's unconservative predictions on high-strength concrete components with low prestress (Fig. 5).

Tables 2 and 3 present verification results of the proposed V_{cw} in Eq. (14) and (15) for all prestressed concrete beam test results with and without shear reinforcement and prestressed concrete hollow-core slabs. The proposed method showed comparable prediction accuracy and conservatism to the current ACI design method irrespective of the presence of shear reinforcement.

The web shear strengths of hollow-core slab test specimens were compared to those estimated by ACI 318-19 and the proposed modification (**Fig. 8**). For hollow-core slab members with h greater than 315 mm (12.5 in.), the aforementioned cut-in-half rule in web shear strength ($V_{cal} = \phi V_{cw}/2$, where ϕ is 1.0) was used. In terms of average, standard deviation, and coefficient of variation (COV), the proposed and current methods were nearly identical. Thus, it can be confirmed that the proposed revision will have no substantial impact on the future hollow-core slab industry.

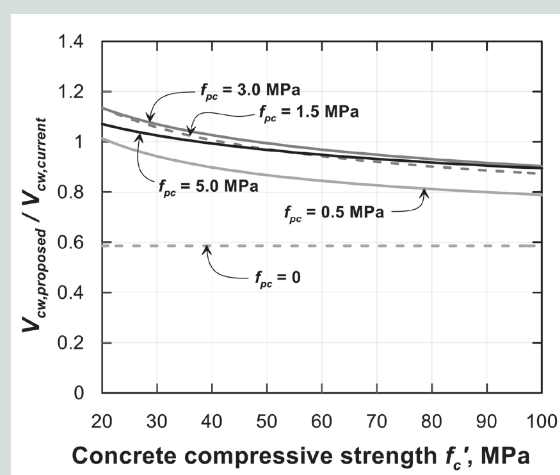

Figure 7. Ratio of web-shear strengths of proposed-to-current methods. Note: f_{ci}' = compressive strength of concrete; f_{pc} = compressive stress in concrete after allowance for all prestress losses at centroid of cross section; $V_{cw,current}$ = web-shear strength predicted by the current ACI 318-19 method; $V_{cw,proposed}$ = web-shear strength predicted by the proposed method. 1 MPa = 0.145 ksi.

Statistical evaluation of proposed modifications

Figure 9 compares shear strengths of test specimens without shear reinforcement with shear strengths calculated using the

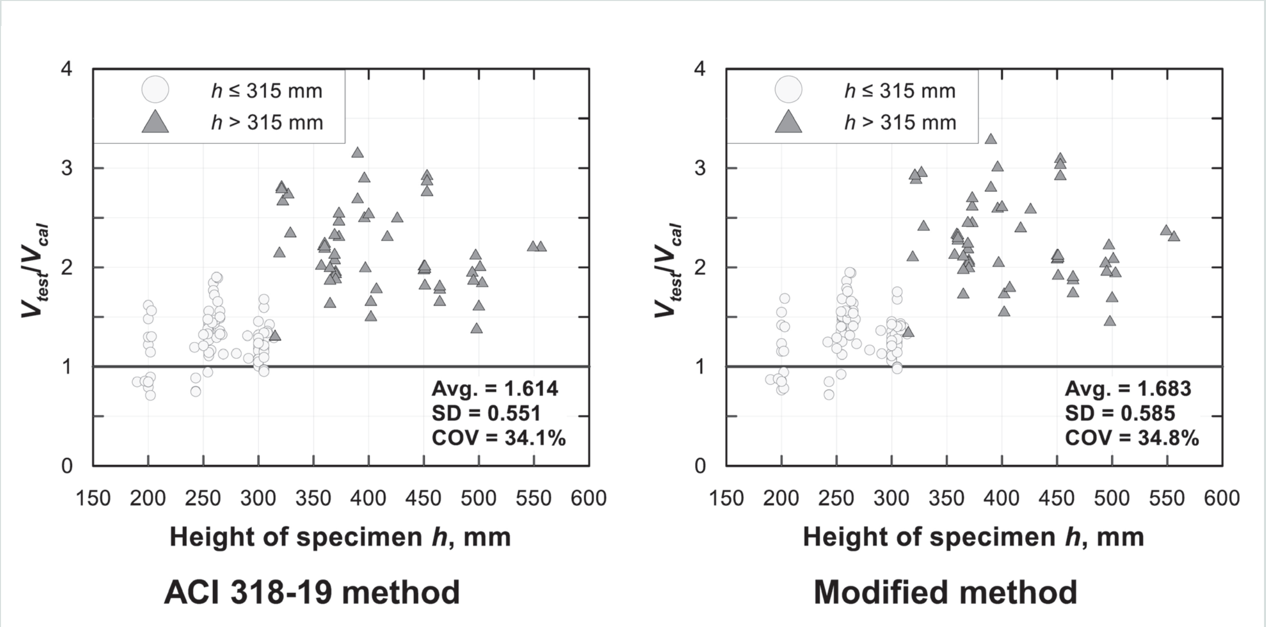

Figure 8. Influence of modification on prestressed hollow-core slab members considering *American Concrete Institute's Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)* section 7.6.3.1's cut-in-half rule. Note: Avg = average; COV = coefficient of variation; SD = standard deviation; V_{cal} = calculated shear strength per ACI 318-19 or proposed detailed method; V_{test} = measured shear strength in shear database. 1 mm = 0.0394 in.

ACI 318-19 detailed method and the proposed method. **Figure 10** provides similar comparisons for specimens with shear reinforcement. The calculated shear strength V_{cal} was determined from the lesser of V_{ci} and V_{cw} for the test specimens. The proposed methodology provided slightly enhanced analytical accuracy in terms of the estimated average, with an average of 1.57 for the test specimens without shear reinforcement and an average of 1.30 for those with shear reinforcement (Tables 2 and 3). Furthermore, the proposed methodology was analytically more accurate for the coefficient of variation, with COV of 33% for the test specimens without shear reinforcement and COV of 20% for those with shear reinforcement. While the percentage of predicted results falling below experimental data becomes slightly unconservative in the proposed method, it is about 2% different from the current ACI 318 method, which would not significantly affect the design results.

The test-to-prediction ratios V_{test}/V_{cal} were also compared with respect to the shear-related parameters f'_c, d, a/d, ρ_{wt}, and f_{pc} (Fig. 9 and 10). There are no significant differences on the V_{test}/V_{cal} distributions between the ACI 318-19 and proposed methods because the proposed method has the same inherent philosophy as the ACI 318-19 provision. However, it can again be confirmed that the underestimation of shear strength for highly reinforced members ($\rho_{wt} \geq 2.0\%$) is somewhat mitigated in the proposed method and gives a more reasonable prediction than the ACI 318-19 method. The primary purpose of this study was to provide a more straightforward shear design methodology for prestressed concrete members. Improved accuracy is considered to be a secondary favorable outcome associated with the proposed modifications.

Evaluation of proposed detailed method through design examples

The results calculated from the proposed detailed shear formula were compared with results calculated from the ACI 318-19 detailed shear provisions for three design examples:

- a hollow-core slab with a straight tendon profile (**Fig. 11**)
- a two-span, post-tensioned tee beam with an idealized parabolic tendon profile (**Fig. 12**)
- a double-tee-shaped precast concrete and cast-in-place composite girder with harped strand profile (**Fig. 13**)

The geometrical and material information for the design examples was adopted from the *PCI Design Handbook*[55] and the *Post-Tensioning Manual*.[56] Information on the cross sections and materials is shown in the figures for the examples. In this paper, only the calculated shear strengths by concrete along the length are compared. For the selected location x, where x is distance from end support, the detailed procedures using proposed modifications are presented.

Example 1: Hollow-core slab

For the hollow-core slab example (Fig. 11), the procedure to calculate V_c by hand at the selected location x is 1000 mm was as follows.

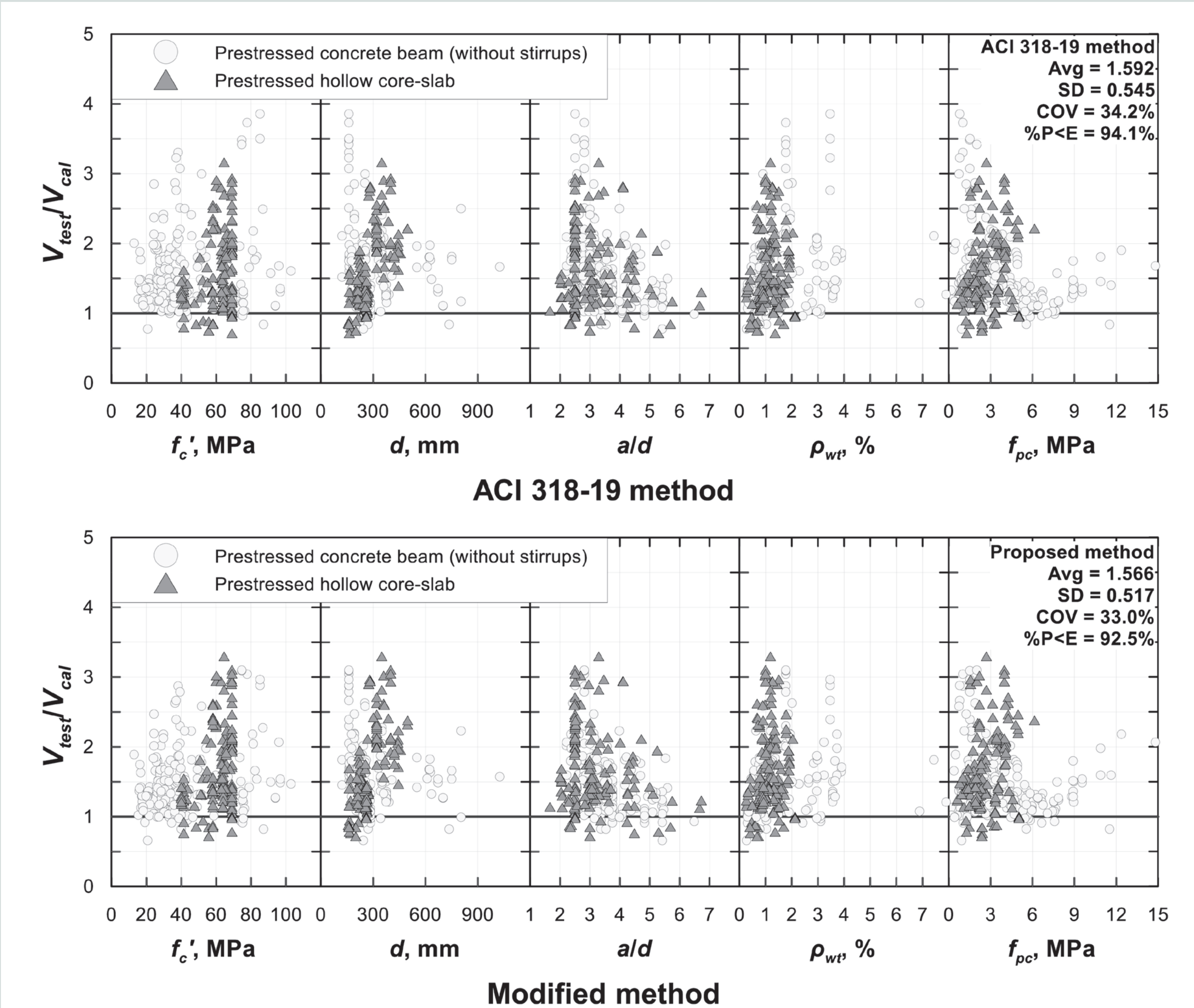

Figure 9. Verification of proposed detailed methods for specimens without shear reinforcement. Note: a = shear span; ACI 318-19 = American Concrete Institute's *Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)*; Avg = average; COV = coefficient of variation; d = effective beam depth; f'_{ci} = compressive strength of concrete; f_{pc} = compressive stress in concrete after allowance for all prestress losses at centroid of cross section; SD = standard deviation; V_{cal} = calculated shear strength per ACI 318-19 or proposed detailed method; V_{test} = measured shear strength in shear database; ρ_{wt} = longitudinal reinforcement ratio; %P<E = percentage of predicted results falling below experimental data of analysis results. 1 mm = 0.0394 in.; 1 MPa = 0.145 ksi.

Demand calculation

w_u = 1.2(1.641 + 0.700) + 1.6(1.800) = 5.689 kN/mm (0.39 kip/ft)

where

w_u = factored distributed load per unit length of component

$$V_u = \left(\frac{L}{2} - x\right)w_u = \left[\left(\frac{7}{2}\right) - 1\right](5.689) = 14.22 \text{ kN } (3.20 \text{ kip})$$

where

L = span length of prestressed member

$$M_u = \left(\frac{L}{2} - \frac{x^2}{2}\right)w_u = \left[\left(\frac{7\times 1}{2}\right) - \frac{1^2}{2}\right](5.689)$$

$$= 17.07 \text{ kN } (12.59 \text{ kip-ft})$$

Flexure shear strength V_{ci}

Since the location x 1000 mm (39.4 in.) was not within the transfer length ℓ_{tr} is 635 mm (25 in.), the full effective prestress force was applied at this location.

ℓ_{tr} = $50d_b$ = 50 × 12.7 = 635 mm (25 in.) < x = 1000 mm (39.4 in.)

where

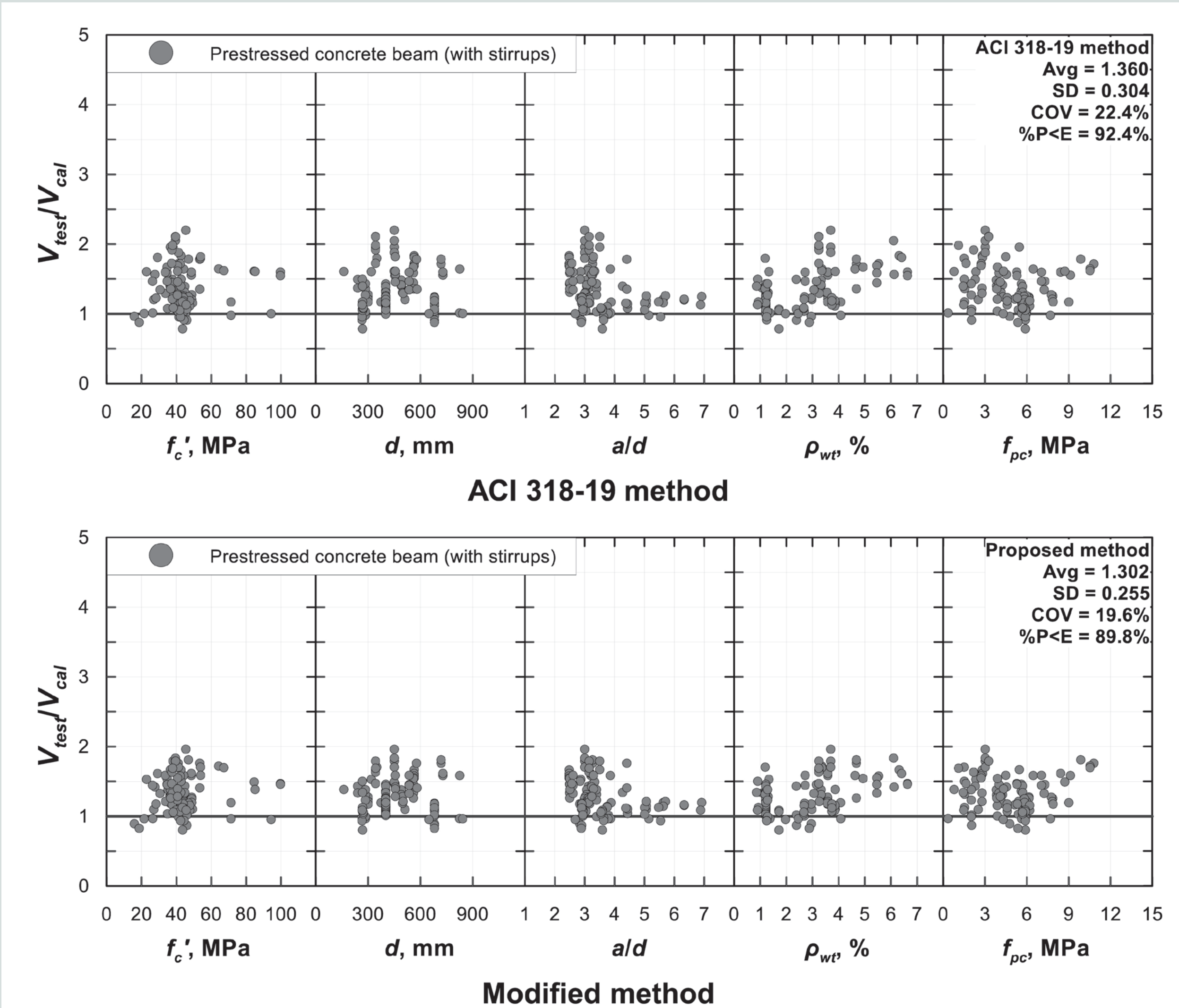

Figure 10. Verification of proposed detailed methods for specimens with shear reinforcement. Note: a = shear span; ACI 318-19 = American Concrete Institute's *Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)*; Avg = average; COV = coefficient of variation; d = effective beam depth; f'_{ci} = compressive strength of concrete; f_{pc} = compressive stress in concrete after allowance for all prestress losses at centroid of cross section; SD = standard deviation; V_{cal} = calculated shear strength per ACI 318-19 or proposed detailed method; V_{test} = measured shear strength in shear database; ρ_{wt} = longitudinal reinforcement ratio; %P<E = percentage of predicted results falling below experimental data of analysis results. 1 mm = 0.0394 in.; 1 MPa = 0.145 ksi.

d_b = nominal diameter of bar, wire, or prestressing strand

$$P_e = A_{ps}f_{se} = 394.8 \times 930 = 367.16 \text{ kN (82.55 kip)}$$

where

P_e = effective prestressing force

$$f_{pc} = \frac{P_e}{A_g} = \frac{367.16\times1000}{66,986} = 5.48 \text{ Mpa } \left(0.795 \text{ ksi}\right)$$

Because d_p 150 mm (5.9 in.) was shorter than 0.8h = 160 mm (6.3 in.) and no longitudinal non-prestressed reinforcement was provided, both d and d_p were taken as 0.8h = 160 mm for detailed method.

$$K = 4\rho_{wt}^{\frac{1}{3}} = 4\left(\frac{A_{ps}}{b_w d}\right)^{\frac{1}{3}} = 4\left[\frac{394.8}{\left(150\times160\right)}\right]^{\frac{1}{3}} = 1.017 > 1.0 \rightarrow K = 1.017$$

$$f_{pe} = \frac{P_e}{A_g} + \frac{P_e e y_b}{I_g} = \frac{367.16\times1000}{66,986} + \frac{367.16\times1000\times\left(150-100\right)\times100}{3.254\times10^8} = 11.12 \text{ Mpa } \left(1.613 \text{ ksi}\right)$$

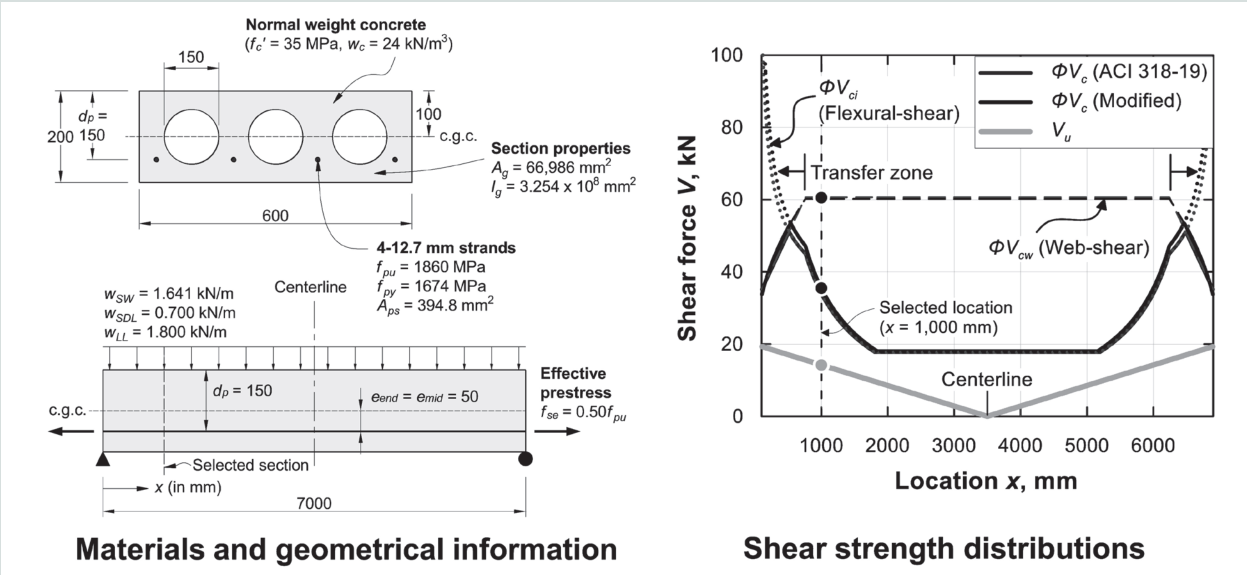

Figure 11. Design example 1: Precast concrete hollow core-slab with straight tendon profile. Note: Dimensions are in millimeters. ACI 318-19 = American Concrete Institute's *Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)*; A_g = area of gross (composite) concrete section; A_{ps} = area of prestressed longitudinal tension reinforcement; c.g.c. = center of gravity of concrete; d_p = distance from extreme compression fiber to prestressed longitudinal reinforcement; f'_{ci} = compressive strength of concrete; f_{pu} = tensile strength of prestressed reinforcement; f_{py} = yield strength of prestressed reinforcement; f_{se} = effective prestress in prestressed reinforcement; I_g = moment of inertia of gross (composite) concrete section about centroidal axis; V_c = nominal shear strength provided by concrete; V_{ci} = flexure-shear strength; V_{cw} = web-shear strength; V_u = factored shear force at section; w_c = unit weight of concrete; w_{LL} = live load per unit length; w_{SDL} = superimposed dead load per unit length; w_{SW} = self-weight of component per unit length; x = distance from end support; e_{end} = eccentricity of prestressed longitudinal reinforcement at the beam end; e_{mid} = eccentricity of prestressed longitudinal reinforcement at the midspan; ϕ = strength reduction factor. 1 mm = 0.0394 in.; 1 mm² = 0.00155 in.²; 1 kN = 0.225 kip; 1 kN/m = 0.0685 kip/ft; 1 MPa = 0.145 ksi.

where

e = eccentricity of prestressed longitudinal reinforcement

I_g = moment of inertia of gross (composite) concrete section about centroidal axis

y_b = distance from centroidal axis to bottom surface of gross (composite) section

$$M_{cr} = \left(\frac{I_g}{y_b}\right)\left(0.62\lambda\sqrt{f_c'} + f_{pe}\right)$$

$$= \left(\frac{3.254\times10^8}{100}\right)\left(0.62\times1.0\times\sqrt{35}+11.12\right)$$

$$= 4.812\times10^7\text{ N-mm} = 48.12\text{ kN-m}\left(35.49\text{ kip-ft}\right)$$

$$V_{ci} = 0.05\lambda K\sqrt{f_c'}b_w d_p + \frac{V_u M_{cr}}{M_u}$$

$$= \frac{0.05\times1.0\times1.017\times\sqrt{35}\times150\times160}{1000}$$

$$+\ \frac{14.22\times48.12}{17.07} = 47.31\text{ kN }\left(10.64\text{ kip}\right)\left(\text{governs}\right)$$

$$V_{ci} \geq 2\lambda K\sqrt{f_c'}b_w d = \frac{0.17\times1.0\times1.017\times\sqrt{35}\times150\times160}{1000}$$

$$= 24.55\text{ kN }\left(5.52\text{ kip}\right)\left(\textbf{OK}\right)$$

Web shear strength V_{cw}

V_p = $P_e \times \theta_p$ = 0 KN (0 kip)

$$V_{cw} = \left(0.17\lambda K\sqrt{f_c'}+\sqrt{f_{pc}}\right)b_w d_p + V_p$$

$$= \left(\frac{0.17\times1.0\times1.017\times\sqrt{35}}{1000}+\frac{\sqrt{5.48}}{1000}\right)\times150\times160+0$$

$$= 80.73\text{ kN }\left(18.15\text{ kip}\right)$$

Shear strength V_c was determined to be the lesser of V_{ci} and V_{cw}. At the selected location x is 1000 mm (39.4 in.), V_{ci} governed the shear strength provided by concrete: $V_c = V_{ci}$ = 47.31 kN (10.64 kip). The V_c along the length was compared with results from the ACI 318-19 detailed method (Fig. 11).

ϕV_{ci} = 0.75 × 47.31 = 35.48 kN (7.98 kip) (governs)

ϕV_{cw} = 0.75 × 80.73 = 60.55 kN (13.61 kip)

ϕV_c = 0.75 × 47.31 = 35.48 kN (7.98 kip)

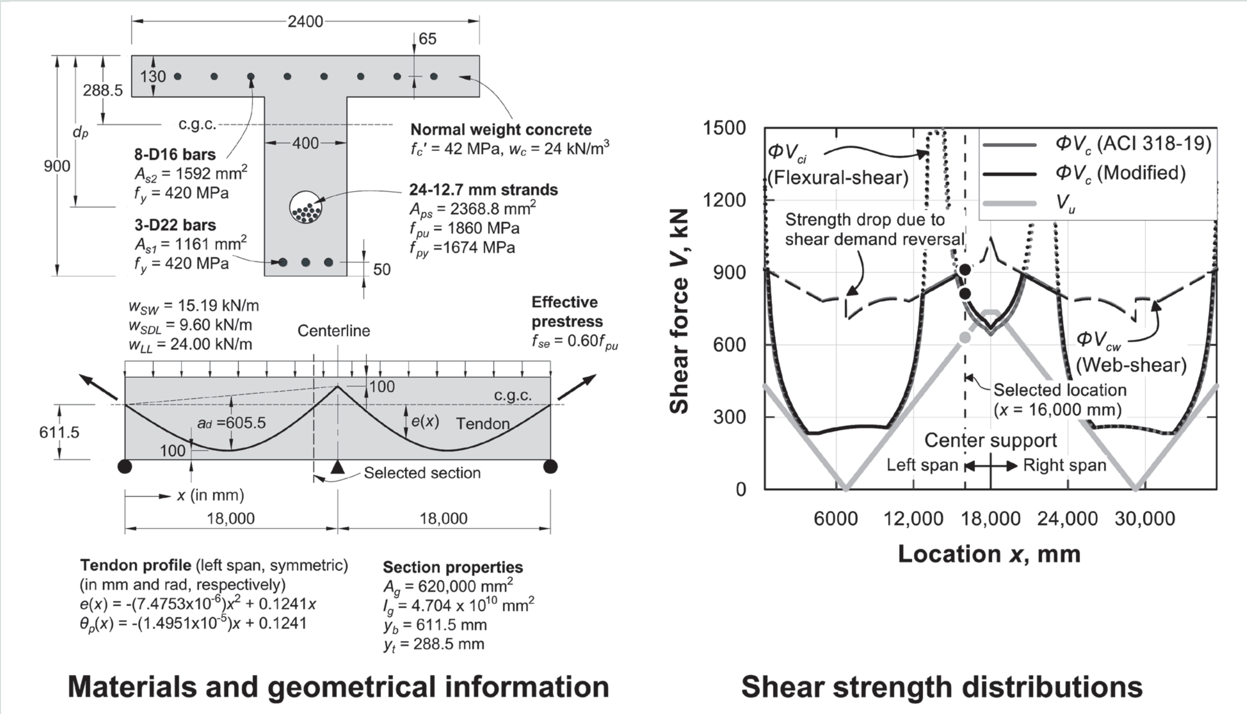

Figure 12. Design example 2: Two span post-tensioned tee beam with idealized parabolic tendon profile. Note: Dimensions are in millimeters. ACI 318-19 = American Concrete Institute's *Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)*; a_d = sag of prestressed longitudinal reinforcement; A_g = area of gross (composite) concrete section; A_{ps} = area of prestressed longitudinal tension reinforcement; A_{s1} = area of nonprestressed bottom longitudinal reinforcement; A_{s2} = area of nonprestressed top longitudinal reinforcement; c.g.c. = center of gravity of concrete; d_p = distance from extreme compression fiber to prestressed longitudinal reinforcement; $e(x)$ = eccentricity of prestressed longitudinal reinforcement (at the location x); f'_{ci} = compressive strength of concrete; f_{pu} = tensile strength of prestressed reinforcement; f_{py} = yield strength of prestressed reinforcement; f_{se} = effective prestress in prestressed reinforcement; V_c = nominal shear strength provided by concrete; V_{ci} = flexure-shear strength; V_{cw} = web-shear strength; V_u = factored shear force at section; w_c = unit weight of concrete; w_{LL} = live load per unit length; w_{SDL} = superimposed dead load per unit length; w_{SW} = self-weight of component per unit length; x = distance from end support; y_b = distance from centroidal axis to bottom surface of gross (composite) section; y_t = distance from centroidal axis to top surface of gross (composite) section; $\theta_p(x)$ = slope of prestressed longitudinal reinforcement (at the location x); ϕ = strength reduction factor. 1 mm = 0.0394 in.; 1 mm² = 0.00155 in.²; 1 kN = 0.225 kip; 1 kN/m = 0.0685 kip/ft; 1 kN/m³ = 0.00637 kip/ft³; 1 MPa = 0.145 ksi.

Example 2: Two-span, post-tensioned tee beam with parabolic tendon profile

For the two-span, continuous post-tensioned tee beam example (Fig. 12), the following procedure was used to hand calculate V_c at the selected location x equal to 16 m (52.5 ft) from the left support. In this example, friction and anchor set loss of tendon were assumed to be zero.

Demand calculation

$$w_u = 1.2(15.19 + 9.60) + 1.6(24.00)$$

$$= 68.148 \text{ kN/m (4.67 kip/ft)}$$

$$V_u = \left(\frac{3L}{8} - x\right) w_u = \left[\left(\frac{3\times18}{8}\right) - 16\right](68.148)$$

$$= -630.37 \text{ kN } (-141.72 \text{ kip})$$

$$M_u = \left(\frac{3Lx}{8} - \frac{x^2}{2}\right) w_u = \left[\left(\frac{3\times18\times16}{8}\right) - \frac{16^2}{2}\right](68.148)$$

$$= -1362.96 \text{ kN-m } (-1005.72 \text{ kip-ft})$$

Flexure shear strength V_{ci}

$$P_e = A_{ps} f_{se} = 2368.8 \times 1116 = 2643.58 \text{ kN (594.33 kip)}$$

$$f_{pc} = \frac{P_e}{A_g} = \frac{2643.58\times1000}{620{,}000}$$

$$= 4.264 \text{ MPa } (0.618 \text{ ksi})$$

Because the component had both prestressing tendon and longitudinal non-prestressed reinforcements, d_p and d were calculated as follows:

$$d_p = e(x) + y_b \text{ (for negative moment region)}$$

$$= [(7.4753 \times 10^{-6})(16{,}000)^2 - 0.1241(16{,}000)] + 611.5$$

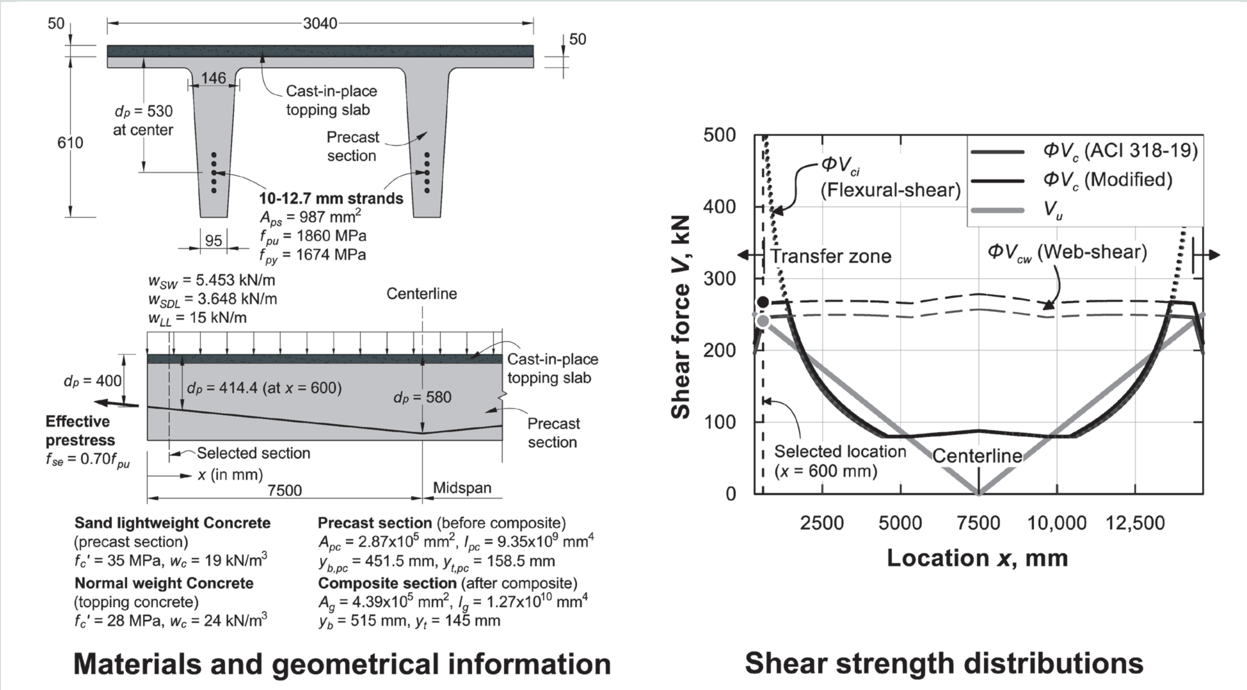

Figure 13. Design example 3: Pretensioned prestressed concrete, cast-in-place concrete composite double tee girder with harped tendon profile. Note: Dimensions are in millimeters. ACI 318-19 = *American Concrete Institute's Building Code Requirements for Structural Concrete* (ACI 318-19) and Commentary (ACI 318R-19); A_g = area of gross (composite) concrete section; A_{pc} = area of precast concrete section only; A_{ps} = area of prestressed longitudinal tension reinforcement; d_p = distance from extreme compression fiber to prestressed longitudinal reinforcement; f'_{ci} = compressive strength of concrete; f_{pu} = tensile strength of prestressed reinforcement; f_{py} = yield strength of prestressed reinforcement; f_{se} = effective prestress in prestressed reinforcement; I_g = moment of inertia of gross (composite) concrete section about centroidal axis; I_{pc} = moment of inertia of precast concrete section only, about the precast concrete section's centroidal axis; V_c = nominal shear strength provided by concrete; V_{ci} = flexure-shear strength; V_{cw} = web-shear strength; V_u = factored shear force at section; w_c = unit weight of concrete; w_{LL} = live load per unit length; w_{SDL} = superimposed dead load per unit length; w_{SW} = self-weight of component per unit length; x = distance from end support; y_b = distance from centroidal axis to bottom surface of gross (composite) section; $y_{b,pc}$ = distance from centroidal axis to bottom surface of precast concrete section; y_t = distance from centroidal axis to top surface of gross (composite) section; $y_{t,pc}$ = distance from centroidal axis to top surface of precast concrete section; ϕ = strength reduction factor. 1 mm = 0.0394 in.; 1 mm² = 0.00155 in.²; 1 mm⁴ = 0.0000024 in.⁴; 1 kN = 0.225 kip; 1 kN/m = 0.0685 kip/ft; 1 kN/m³ = 0.00637 kip/ft³; 1 MPa = 0.145 ksi.

$$= 539.58 \text{ mm (21.24 in.)}$$

$$d = \frac{A_s f_y \left(h - d_s\right) + A_{ps} f_{py} d_p}{A_s f_y + A_{ps} f_{py}}$$

$$= \frac{1592 \times 420 \times \left(900 - 65\right) + 2368.8 \times 1674 \times 539.58}{1592 \times 420 + 2368.8 \times 1674}$$

$$= 582.21 \text{ mm } \left(22.92 \text{ in.}\right)$$

The calculated d and d_p (582.21 and 539.58 mm [22.92 and 21.24 in.], respectively) were less than $0.8h$ equal to 720 mm (28.35 in.). Both d and d_p were taken as $d = d_p = 720$ mm for the detailed method.

$$K = 4\rho_{wt}^{\frac{1}{3}} = 4\left[\left(A_{ps} + A_s\right)/b_w d\right]^{\frac{1}{3}}$$

$$= 4\left[\left(2368.8 + 1592\right)/\left(400 \times 750\right)\right]^{\frac{1}{3}}$$

$$= 0.9583 < 1.0 \rightarrow K = 1.0$$

Because the structure was statically indeterminate, secondary force at the center support was counted in the concrete stress due to prestress f_{pe}. In this case, the balanced moment M_{bal} was not the same as $P_e e$.

$$w_{p,eq} = -\frac{8P_e a_d}{L^2} = -\frac{8 \times 2643.58 \times 605.5}{18{,}000^2}$$

$$= -39.523 \text{ kN/m } \left(-2.71 \text{ kip/ft}\right)$$

where

a_d = sag of prestressed longitudinal reinforcement

$w_{p,eq}$ = equivalent distributed load due to effective prestress

$$M_{bal} = \left(\frac{3Lx}{8} - \frac{x^2}{2}\right) w_{p,eq} = \left(\frac{3 \times 18 \times 16}{8} - \frac{16^2}{2}\right)\left(-39.523\right)$$

$$= 790.46 \text{ kN-m } \left(583.04 \text{ kip-ft}\right)$$

$$f_{pe} = \frac{P_e}{A_g} - \frac{M_{bal}y_t}{I_g} = \frac{2643.58\times1000}{620{,}000} + \frac{790.46\times10^6\times288.5}{4.704\times10^{10}}$$

$$= 9.112 \text{ MPa } (1.32 \text{ ksi})$$

$$M_{cr} = \left(\frac{I_g}{y_t}\right)\left(0.62\lambda\sqrt{f_c'} + f_{pe}\right)$$

$$= \left(\frac{4.704\times10^{10}}{288.5}\right)\left(0.62\times1.0\times\sqrt{42} + 9.112\right)$$

$$= 2140.86 \text{ kN-m } (1579.40 \text{ kip-ft})$$

$$V_{ci} = 0.05\lambda K\sqrt{f_c'}b_w d_p + \frac{V_u M_{cr}}{M_u}$$

$$= \frac{0.05\times1.0\times1.0\times\sqrt{42}\times400\times720}{1000} + \frac{630.37\times2140.86}{1362.96}$$

$$= 1083.47 \text{ kN } (24.59 \text{ kip})$$

$$V_{ci} \ge 0.17\lambda K\sqrt{f_c'}b_w d = \frac{0.17\times1.0\times1.0\times\sqrt{42}\times400\times720}{1000}$$

$$= 317.30 \text{ kN } (71.34 \text{ kip})$$

Web shear strength V_{cw}

$$\theta_p(x) = \frac{d}{dx}e(x) = 2\times\left(7.4753\times10^{-6}\right)x - 0.1241$$

$$= 2\times\left(7.4753\times10^{-6}\right)(16{,}000) - 0.1241 = 0.115$$

$$V_p = P_e \times [-\theta_p(x) \times \text{sign}(V_u)] = 2643.58 \times [-0.1151 \times (-1)]$$

$$= 304.28 \text{ kN } (68.41 \text{ kip})$$

$$V_{cw} = \left(0.17\lambda K\sqrt{f_c'} + \sqrt{f_{pc}}\right)b_w d_p + V_p$$

$$= \left(\frac{0.17\times1.0\times1.0\times\sqrt{42}}{1000} + \frac{\sqrt{4.264}}{1000}\right)\times400\times720 + 304.28$$

$$= 1216.28 \text{ kN } (273.44 \text{ kip})$$

The V_c was determined to be the lesser of V_{ci} and V_{cw}. At the selected location x equal to 16 m (52.5 ft), V_{ci} governed the shear strength provided by concrete: $V_c = V_{ci} = 1083.47$ kN (243.59 kip). Figure 12 compares the V_c along the length with the ACI 318-19 detailed method.

ϕV_{ci} = 0.75 × 1083.47 = 812.60 kN (182.69 kip) (governs)

ϕV_{cw} = 0.75 × 1216.28 = 912.21 kN (205.08 kip)

ϕV_c = 0.75 × 1083.47 = 812.60 kN (182.69 kip)

Example 3: Double-tee-shaped prestressed-to-cast-in-place concrete composite girder with harped strand profile

For the pretensioned double-tee composite girder example (Fig. 13), the procedure to hand calculate V_c at the selected location x equal to 600 mm (23.6 in.) (within the transfer length) was as follows.

Demand calculation

w_u = 1.2(5.453 + 3.648) + 1.6(15.000) = 34.921 kN/m (2.39 kip/ft)

w_d = 5.453 + 3.648 = 9.101 kN/m (0.62 kip/ft)

$$V_u = \left(\frac{L}{2} - x\right)w_u = \left[\left(\frac{15}{2}\right) - 0.6\right](34.921)$$

$$= 240.94 \text{ kN } (54.17 \text{ kip})$$

$$M_u = \left(\frac{Lx}{2} - \frac{x^2}{2}\right)w_u = \left[\left(\frac{15\times0.6}{2}\right) - \frac{0.6^2}{2}\right](34.921)$$

$$= 150.86 \text{ kN-m } (111.27 \text{ kip-ft})$$

$$M_d = \left(\frac{Lx}{2} - \frac{x^2}{2}\right)w_d = \left[\left(\frac{15\times0.6}{2}\right) - \frac{0.6^2}{2}\right](9.101)$$

$$= 39.32 \text{ kN-m } (29.0 \text{ kip-ft})$$

where

M_d = moment at section due to unfactored dead load

w_d = unfactored dead load per unit length of component

Flexure shear strength V_{ci}

Since the location x equal to 600 mm (23.6 in.) was within the transfer length ℓ_{tr} equal to 635 mm (25 in.), the reduced effective prestress force was applied at this location.

ℓ_{tr} = $50d_b$ = 50 × 12.7 = 635 mm (25 in.)

> x = 600 mm (23.6 in.)

P_e = $(x/l_{tr})\,A_{ps}f_{se}$ = (600/635) × 987 × 1302

= 1214 kN (272.93 kip)

$$f_{pc} = \frac{P_e}{A_{pc}} + \left(M_d - P_e e_{pc}\right)\frac{y_b - y_{b,pc}}{I_{pc}}$$

$$= \frac{1.285\times10^6}{2.87\times10^5} + \left(39.32\times10^6 - 1.285\times10^6\times205.9\right)\times\frac{515.0-451.5}{9.35\times10^9}$$

$$= 2.948 \text{ MPa } (0.428 \text{ ksi})$$

where

e_{pc} = eccentricity of prestressed longitudinal reinforcement with respect to centroidal axis of precast concrete section

$y_{b,pc}$ = distance from centroidal axis to bottom surface of precast concrete section

I_{pc} = moment of inertia of precast concrete section only, about the precast concrete section's centroidal axis

Because d_p equal to 414.4 mm was smaller than 0.8h equal to 528 mm, both d and d_p were taken as 0.8h equal to 528 mm for the detailed method.

$$K = 4\rho_{wt}^{1/3} = 4\left[\left(A_{ps}+A_s\right)/b_w d\right]^{1/3}$$
$$= 4\left[\left(2368.8+1592\right)/\left(400\times750\right)\right]^{1/3}$$
$$= 0.9583 < 1.0 \rightarrow K = 1.0$$

$$f_{pe} = \frac{P_e}{A_{pc}} + \frac{P_e e_{pc} y_{b,pc}}{I_{pc}}$$
$$= \frac{1.214\times10^6}{2.87\times10^5} + \frac{1.214\times10^6\times205.9\times451.5}{9.35\times10^9}$$
$$= 16.30 \text{ MPa } \left(2.36 \text{ ksi}\right)$$

where

A_{pc} = area of precast concrete section only

$$M_{cr} = \left(\frac{I_g}{y_b}\right)\left(0.62\lambda\sqrt{f_c'} + f_{pe}\right)$$
$$= \left(\frac{1.2728\times10^{10}}{515}\right)\left(0.62\times0.85\times\sqrt{35}+16.30\right)$$
$$= 479.90 \text{ kN-m } \left(353.97 \text{ kip-ft}\right)$$

$$V_{ci} = 0.05\lambda K\sqrt{f_c'}b_w d_p + \frac{V_u M_{cr}}{M_u}$$
$$= \frac{0.05\times0.85\times1.0\times\sqrt{35}\times241\times528}{1000} + \frac{240.95\times479.90}{150.86}$$
$$= 798.48 \text{ kN } \left(179.51 \text{ kip}\right)$$

$$V_{ci} = 0.17\lambda\kappa\sqrt{f_c'}b_w d = \frac{0.17\times0.85\times1.0\times\sqrt{35}\times241\times528}{1000}$$
$$= 108.78 \text{ kN } \left(24.46 \text{ kip}\right)\left(\textbf{OK}\right)$$

Web shear strength V_{cw}

$$V_p = P_e\times\theta_p \text{ ; } P_e\frac{e_{mid}-e_{end}}{L/2} = 1214\times\frac{371.5-191.5}{7500}$$
$$= 29.14 \text{ kN } \left(6.55 \text{ kip}\right)$$

where

e_{end} = eccentricity of prestressed longitudinal reinforcement at the beam end

e_{mid} = eccentricity of prestressed longitudinal reinforcement at the midspan

$$V_{cw} = \left(0.17\lambda K\sqrt{f_c'} + \sqrt{f_{pc}}\right)b_w d_p + V_p$$
$$= \left(\frac{0.17\times0.85\times1.0\times\sqrt{35}}{1000} + \frac{\sqrt{2.948}}{1000}\right)\times241\times528+29.14$$
$$= 356.40 \text{ kN } \left(80.13 \text{ kip}\right)\left(\text{governs}\right)$$

Shear strength V_c was determined to be the lesser of V_{ci} and V_{cw}. At the selected location x equal to 600 mm (23.6 in.), V_{cw} governed the shear strength provided by concrete: $V_c = V_{cw}$ = 356.40 kN (80.13 kip). Figure 13 compares the V_c along the length with results from the ACI 318-19 detailed method.

ϕV_{ci} = 0.75 × 798.48 = 598.86 kN (134.64 kip)

ϕV_{cw} = 0.75 × 356.40 = 267.30 kN (60.09 kip) (governs)

ϕV_c = 0.75 × 356.40 = 267.30 kN (60.09 kip)

Comparison between current and proposed methods

Shear strengths determined by the ACI 318-19 detailed method and the proposed modifications were nearly identical in examples 1 and 2 (Fig. 11 and 12). Because the longitudinal reinforcement ratios ρ_{wt} in those cases were less than 1.56%, the introduction of the K factor had no effects on shear strength. The other influential factor, dead load V_d, also had little effect in these examples. The examples confirmed that excluding the dead load term has limited effects on V_{ci}. In the case of V_{cw}, the terms in parentheses of Eq. (8) and (9) and Eq. (14) and (15) were coincidentally close to each other and slight differences (less than 1%) in web shear strength were observed. In example 2 (Fig. 12), the V_{cw} value was discontinuous at the center of the span because the direction of applied shear force at that point was reversed and the sign of V_p became opposite. Consequently, the proposed modifications had no substantial effects on the components' shear design results.

In example 3, the values of V_{ci} were essentially the same, whereas the V_{cw} obtained from the modified formula was estimated to be 8% higher than that of the ACI 318-19 method. With higher strength of concrete, the predictions were more conservative with the proposed formula If the concrete compressive strength f_c' was 49 MPa (7 ksi) or higher, the proposed method always predicted a lower V_{cw} than the current detailed formula when K is 1.0. This can be attributed to the relatively low concrete strength given in this example. There was a slight design change only near the supports where governed by web shear (Fig. 13).

As shown in these design examples, the proposed modifications were easier to calculate, but the final component design was minimally affected.

Conclusion

Attributed to various interrelated influential factors, the shear resistance mechanism in a prestressed concrete component is difficult to understand and eludes common consensus regarding general shear design methodologies.

The one-way shear provision in ACI 318-19 has been little changed since being introduced in the 1971 edition of ACI 318. Previous studies have criticized its complicated calculation procedure, especially with prestressed concrete members. Furthermore, the basis for shear design methods is based on available past shear test data. Current shear design provisions also need to address post-tensioned members. In that sense, there has been an ongoing need to improve the prestressed concrete shear provisions, just as the reinforced concrete design method was revised in ACI 318-19.

This study aimed to make reasonable modifications to the existing detailed approach in ACI 318 for prestressed concrete components of building structures in keeping with the inherent philosophy and framework of current shear design provisions. Proposed modifications focused on developing more simple and intuitive computational procedures with commensurate analytical accuracy to current shear design formulas.

In the proposed modifications, the flexure shear strength V_{ci} equation for the detailed prestressed concrete shear provisions of ACI 318-19 was adjusted by eliminating the effect of the dead load. Then, the factored moment M_{max} and shear force V_i occurring simultaneously due to externally applied load were integrated into the factored moment M_u and shear force V_u, which made the demand calculation more straightforward. Finally, the effect of tensile reinforcement ratio ρ_{wt} was addressed with the introduction of coefficient K. For the web shear strength V_{cw}, concerns regarding abrupt strength jump and dependency on f_{pc} were rationalized by proposed modifications based on current shear database.

Proposed equations for V_c (Eq. [12] and [13]) and V_{cw} (Eq. [14] and [15]) were verified by using the current shear databases for various prestressed concrete beams and hollow-core slabs, and results were compared with those from the detailed formulas in ACI 318-19. When averages and COVs were calculated using proposed methods in terms of test-to-predicted shear strength ratio V_{test}/V_{cal}, they were close to those obtained from ACI 318-19 methods. The rationality of the proposed modifications was reconfirmed in prestressed member design examples, including three presented in this paper.

Proposed modifications on prestressed concrete shear provide efficient and accurate computation with reduced analytical derivation. The authors hope this proposal will be helpful to improve upon the ACI 318-19 prestressed concrete shear design provisions.

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Notation

a = shear span

a_d = sag of prestressed longitudinal reinforcement

A_g = area of gross (composite) concrete section

A_{pc} = area of precast concrete section only

A_{ps} = area of prestressed longitudinal tension reinforcement

As = area of nonprestressed longitudinal tension reinforcement

A_{s1} = area of nonprestressed bottom longitudinal reinforcement

A_{s2} = area of nonprestressed top longitudinal reinforcement

A_v = area of shear reinforcement within a distance s, where s is spacing of transverse reinforcement

$A_{v,min}$ = area of minimum shear reinforcement

b' = minimum width of web of a flanged component according to ACI 318-63

b_w = web width of component

d = effective beam depth (in other words, the distance from the extreme compression fiber to centroid of longitudinal tension reinforcement, defined based on the centroid of resultant tension force, which need not be less than $0.8h$, Eq. [3])

d_b = nominal diameter of bar, wire, or prestressing strand

d_p = distance from extreme compression fiber to prestressed longitudinal reinforcement, which need not be less than $0.8h$ when applied to the current and proposed detailed methods (Eq. [4], [5], [8], [9], [12], [13], [14], and [15])

d_s = distance from extreme compression fiber to centroid of nonprestressed longitudinal reinforcement

e = eccentricity of prestressed longitudinal reinforcement (at the location x)

$e(x)$ = eccentricity of prestressed longitudinal reinforcement (at the location x)

e_{end} = eccentricity of prestressed longitudinal reinforcement at the beam end

e_{mid} = eccentricity of prestressed longitudinal reinforcement at the midspan

e_{pc} = eccentricity of prestressed longitudinal reinforcement with respect to centroidal axis of precast concrete section

f'_c = compressive strength of concrete

f_d = compressive stress due to unfactored dead load at extreme fiber of section where tensile stress is caused by externally applied loads

f_{pc} = compressive stress in concrete after allowance for all prestress losses at centroid of cross section resisting externally applied loads or at junction of web and flange where the centroid lies within the flange. For pretensioned components, reduced effective prestress should be considered by taking ℓ_{tr} equal to $50d_b$ as transfer length. In a composite component, f_{pc} is the resultant compressive stress at centroid of composite section, or at junction of web and flange where the centroid lies within the flange, due to both prestress and moments resisted by precast concrete component acting alone.

f_{pe} = compressive stress in concrete due to effective prestress forces only

f_{pu} = tensile strength of prestressed reinforcement

f_{py} = yield strength of prestressed reinforcement

f_{se} = effective prestress in prestressed reinforcement

f_y = yield strength of nonprestressed longitudinal reinforcement

f_{yt} = yield strength of transverse reinforcement

h = member height or thickness

I = moment of inertia of section about centroidal axis

I_g = moment of inertia of gross (composite) concrete section about centroidal axis

I_{pc} = moment of inertia of precast concrete section only, about the precast concrete section's centroidal axis

ℓ_{tr} = transfer length of pretensioned member

K = longitudinal reinforcement coefficient

L = span length of prestressed member

M = bending moment due to externally applied load, according to ACI 318-63

M_{bal} = balanced moment

M_{cr} = cracking moment

M_{cre} = moment causing flexural cracking due to externally applied loads

M_d = moment at section due to unfactored dead load

M_{max} = maximum factored moment at section due to externally applied loads

M_u = factored flexural moment

N_u = factored flexural moment

P_e = effective prestressing force = $f_{se}A_{ps}$

V = shear force due to externally applied load, according to ACI 318-63

V_c = nominal shear strength provided by concrete

V_{cal} = calculated shear strength per ACI 318-19 or proposed detailed method

V_{ci} = nominal shear strength provided by concrete where diagonal cracking results from combined shear and moment (flexure-shear strength)

V_{cw} = nominal shear strength provided by concrete where diagonal cracking results from high principal tensile stress in web (web-shear strength)

$V_{cw,current}$ = web-shear strength predicted by the current ACI 318-19 method

$V_{cw,proposed}$ = web-shear strength predicted by the proposed method

V_d = shear force at section due to unfactored dead load

V_i = factored shear force at section due to externally applied loads occurring simultaneously with M_{max}

V_n = nominal shear strength

V_p = vertical component of effective prestress force

V_s = nominal shear strength provided by shear reinforcement, simply taken as A_vF_{ytd}/s based on 45-degree truss mode, where s is spacing of transverse reinforcement

V_{test} = measured shear strength in shear database

V_u = factored shear force at section

w_c = unit weight of concrete

w_d = unfactored dead load per unit length of component

w_{LL} = live load per unit length

$w_{p,eq}$ = equivalent distributed load due to effective prestress

w_{SDL} = superimposed dead load per unit length

w_{SW} = self-weight of component per unit length

w_u = factored distributed load per unit length of component

x = distance from end support

y_b = distance from centroidal axis to bottom surface of gross (composite) section

$y_{b,pc}$ = distance from centroidal axis to bottom surface of precast concrete section

y_t = distance from centroidal axis to top surface of gross (composite) section

$y_{t,pc}$ = distance from centroidal axis to top surface of precast concrete section

β = concrete contribution factor

β_{flex} = ratio between shear strength and shear force estimated at the flexural strength

ε_s = strain of longitudinal tensile reinforcement

ε = angle of diagonal compression field

ε_p = slope of prestressed longitudinal reinforcement (at the location x)

$\theta_p(x)$ = slope of prestressed longitudinal reinforcement (at the location x)

$\theta_{p,end}$ = slope of prestressed longitudinal reinforcement at beam-end

$\theta_{p,mid}$ = slope of prestressed longitudinal reinforcement at midspan

λ = lightweight concrete factor

λ_s = size effect factor

ρ_{pw} = prestressed longitudinal reinforcement ratio, defined as A_{ps}/b_wd

ρ_w = non-prestressed longitudinal reinforcement ratio, defined as A_s/b_wd

ρ_{wt} = longitudinal reinforcement ratio, defined as $\rho_w + \rho_{pw}$

ρ_v = transverse reinforcement ratio, defined as $A_v/b_w s$

ϕ = strength reduction factor

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Abstract

This paper proposes modifications to the methods for shear design of prestressed concrete one-way members specified in American Concrete Institute's *Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)* to increase applicability. The current ACI 318 shear design methods have been widely used and have a history of demonstrated safety and reliability. However, there are long-standing concerns regarding the cumbersome computational procedures specified in ACI 318, as well as the inability of the ACI 318 shear design methods to capture key influential factors. This paper provides a brief history of changes made over several decades within ACI 318 for prestressed concrete shear design and critical issues raised in previous studies. While maintaining the philosophy and safety priorities of the original pioneers in the development of shear design for prestressed concrete members, the proposed changes simplify the calculation process and provide analytical accuracies comparable to the current ACI 318 methods. These changes are affirmed by comparing the results of the modified methods with data from an extensive shear database of prestressed concrete component designs that vary in dimensional detail and material properties and shear strengths estimated using current ACI 318 methods.

Keywords

Detailed method, modification, prestressed concrete one-way member, shear design, shear strength.

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